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A Phase-Locked Receiving Array
for High-Frequency Communications Use

by

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ABSTRACT

An H.F. receiving array using I.F. combination and automatic phasing through phase-locked loops is discussed. The effects of an interfering signal on automatic phasing and on the overall array performance are considered. An experimental array is described and results of measurement on the array are given.

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CHAPTER I
INTRODUCTION

The investigation to be described was undertaken in order to determine the feasibility of an automatically phased H. F. receiving array. The array ~~to be considered~~ uses separate R. F. amplifiers for each element. The signals from each element are combined at an intermediate frequency. It is shown that ~~As will be shown,~~ this type of array corrects for essentially all phase errors between the distant transmitter and the point where the received signals are combined. This includes errors due to the propagation path, array element motion, near field obstructions, and instabilities in electronic equipment and R. F. cables. The array also corrects for phase shifts due to changes in angle of arrival, thus giving it the highly desirable property of automatically tracking a desired signal.

CHAPTER II

THE PHASE-LOCKED-ARRAY CONCEPT

For improved reception and interference rejection in the H. F. (3-30 mc) communication region of the spectrum, it is frequently desirable to use a high-gain receiving antenna system. Due to the necessarily large size of an array at these frequencies it is usually impractical to use mechanical positioning of the antenna to scan the beam. A more practical method of obtaining a high-gain steerable beam antenna system is to use an array of fixed omnidirectional elements and form a beam in the desired direction by combining the signals received by each of the elements with the proper amplitude and phase. Amplitude and phase adjustment can be accomplished by the insertion of a variable delay line or phase shifter and an attenuator in the R. F. transmission line between each of the elements and the point at which the R. F. signals are to be combined.

A system using R. F. combination has a number of disadvantages. The long runs of R. F. transmission line from each of the elements are likely to introduce a considerable loss in signal strength. In addition, if other electronic equipment or power equipment is to be operated in the vicinity of the array, interference could be picked up by the transmission lines. Besides being bulky, the required delay lines or phase shifters would introduce an additional loss in signal strength and further reduce the signal-to-noise ratio at the combining point.

These apparent disadvantages can be overcome by the use of R. F. amplification at each of the elements. The amplifier gain would overcome the transmission line and delay line loss and the signal would be transmitted at a level sufficiently high as to cause any interference picked up on the transmission lines to be negligible. Amplitude control could be easily obtained by controlling the R. F. amplifier gain.

The methods of phase adjustment require some further discussion. The necessary phase shift for each of the elements may be obtained by the use of a mechanically adjusted lumped constant delay line or possibly an electronically adjusted delay line using varactor diodes as variable reactance elements. An alternate method, however, is to convert each of the R. F. signals to an intermediate frequency before the signals are combined. In this case phase control may be obtained by using a voltage-tunable local oscillator for the down conversion.

Since the rate of change of phase of a voltage controlled oscillator is proportional to the frequency deviation from the center frequency, it is also proportional to the control voltage. In order to obtain the proper phase control it is simply necessary to insure that the control voltage is proportional to the derivative of the desired phase shift. This can be accomplished by using a closed-loop phase-control system, that is, a phase-locked loop.

A minor disadvantage that should be mentioned is that this system produces a constant phase shift regardless of frequency instead of the time delay a broadband array would require; therefore, an array using it is essentially a single-frequency system. However, since the percentage bandwidth of an H. F. communication channel is normally small, if the array is properly phased at the center of the pass-band the phase error across the passband will be small.

Up to this point nothing has been said about what the proper amplitude and phase distribution over the array should be in terms of the array configuration, which may be dictated by purely mechanical considerations, such as the type of structure on which the array is to be mounted, the angle of arrival of the desired signal and possibly the angles of arrival of a number of undesirable signals. An obvious criterion for amplitude and phase adjustment would be to use that amplitude and phase distribution which would optimize the signal-to-noise ratio at the combining point.

This criterion could conceivably lead to an array which, under some circumstances, would produce a pattern with nulls in the directions of interfering signals rather than a highly directive beam.

It would be highly desirable that amplitude and phase adjustments be done continuously and automatically. The sort of optimization suggested above would be extremely difficult to accomplish since not only would it be necessary to be able to distinguish the desired signal from interference and noise, but it would also be necessary to make provisions for measuring the effects of amplitude and phase adjustments at each of the elements on the signal-to-noise ratio at the combining point.

An alternate criterion for optimization of signal-to-noise ratio can be obtained by assuming the noise introduced by each element and its associated receiving channel to be statistically independent. While this assumption is not very realistic for an H. F. receiving system it does lead to a far easier method of amplitude and phase adjustment which is useful in a practical array.

By considering a simple array of two elements with the assumption of independent noise in each channel it can be shown (see Appendix A) that to optimize the signal-to-noise ratio the signals in each channel should be combined in phase and the relative channel gains should be adjusted according to the relation

$$\text{(A-7)} \qquad \frac{a_2}{a_1} = \frac{A_2}{A_1} \ \frac{N_1}{N_2}$$

where a_1 and a_2 are the voltage gains of channels 1 and 2, respectively, and A_1 and A_2 are the signal amplitudes and N_1 and N_2 are the noise powers referred to the inputs of channels 1 and 2, respectively.

The above results can be extended to any number of elements simply by adding elements to the array one at a time.

In the case where the signal-to-noise ratio for each channel is the same the result would be an array with the far-field pattern expected from a conventional array with uniform amplitude distribution. If an unwanted signal were present in the far field the rejection of this signal could be predicted from the array pattern.

A suggested array phase-and-amplitude-optimizing system is shown in the block diagram of Fig. 1. The desired signal will be assumed to consist of a pilot carrier and information side bands. Each element has its own preselector, R. F. amplifier, and down-converter located at the element. The mixers are all fed from a common local oscillator.

The operating frequency of the array is adjusted by tuning the common local oscillator and each of the preselectors. Phase and amplitude adjustment circuits for each channel can then be operated at a single intermediate frequency. Amplitude adjustment is provided by a gain control voltage which is fed to the I. F.

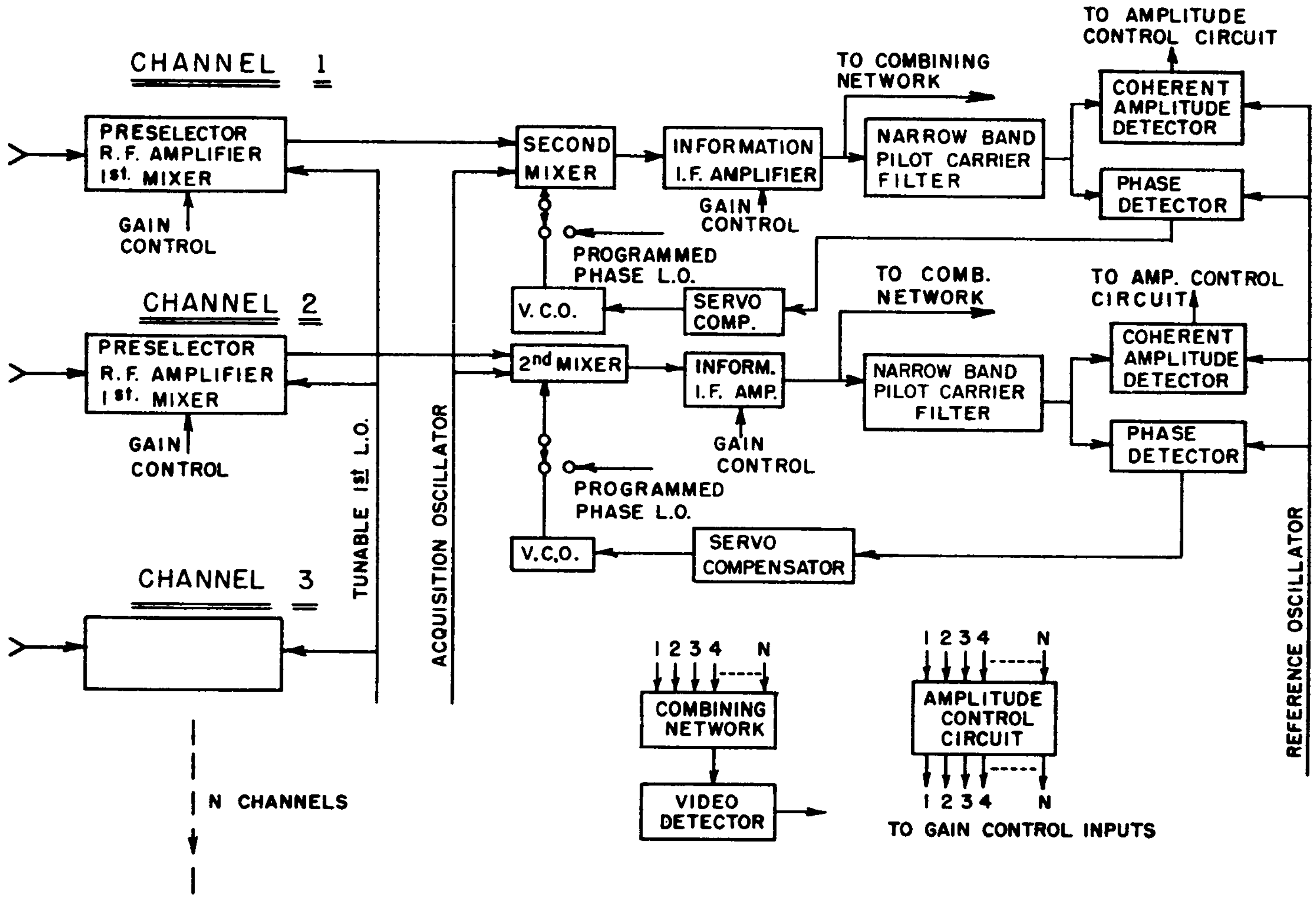


Fig. 1. Block diagram of a phase-locked array.

amplifiers in each channel. Phase adjustment is obtained by using separate voltage controlled local oscillators for a second down-conversion.

The phase-control information is obtained by using a phase detector to compare the carrier phase at the output of each of the second I. F. amplifiers with a reference oscillator which is common to all channels. The phase-error information for each channel is fed back through a servo-compensator to the voltage-controlled second local oscillator (V. C. O.) for phase correction. The phase detector consists essentially of a limiter and a balanced mixer and gives an output proportional to the sine of the phase difference between the signal and reference oscillator. If the phase difference is small the sine function can be linearized and the phase-locked loop can be analyzed as a linear servo system with the input being the signal phase and the output the V. C. O. phase as shown in Figs. 2 (a) and (b).

The transfer function of the loop can be determined in terms of the transfer functions of the various blocks and is given by

(1) $$\frac{\phi_{out}}{\phi_{in}} = \frac{K_{vco} K_{\phi} G(s)}{s + K_{vco} K_{\phi} G(s)}$$

where ϕ_{out} is the output phase, ϕ_{in} is the input phase, K_{vco} is the V. C. O. gain in $\frac{\text{rad.}}{\text{sec-volt}}$, K_{ϕ} is the phase detector gain in volts/rad.,

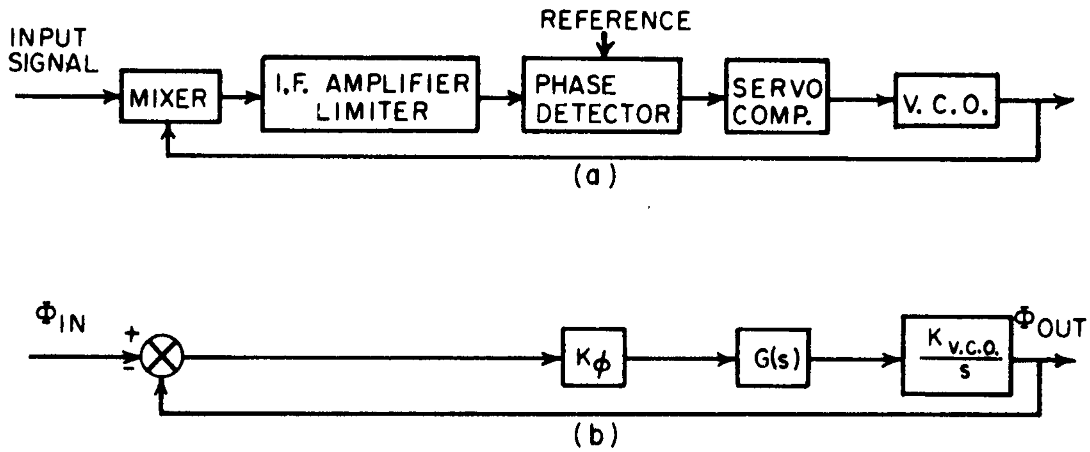


Fig. 2. a) Phase-locked loop. b) Servo representation of the phase-locked loop.

G(s) is the compensator transfer function and s is the Laplace transform variable. The disturbance caused to the pilot carrier by modulation and noise can be thought of as phase noise. The compensator transfer function G(s) is chosen to give proportional-plus-integral compensation. In addition to causing the loop to act as a low-pass filter this choice eliminates any phase error that might be caused by a steady-state frequency offset.

Due to the low-pass filter action, the V.C.O. phase does not follow the input phase perfectly. The low-pass filter action removes much of the phase noise and reduces V. C. O. jitter about the desired phase. Since some jitter is inevitable, perfect phasing cannot be obtained and some deterioration in performance can be expected. It can be shown, for example, that a single sinusoidal interfering signal results in a reduction in array gain and also in amplitude and phase modulation of the desired signal at harmonics of the frequency difference between the interfering signal and the desired pilot carrier (see Appendix B). Deterioration of performance is minimized by using a narrow-band filter preceeding the phase detector to pass the pilot carrier and eliminate much of the noise and interference.

When the pilot carrier of the signal appearing in each channel has been corrected to be in phase with the reference oscillator, a portion of each of the signals can be picked off just before each of the narrow-band filters and combined in phase in a summing network as shown in the block diagram. Signal strength information for amplitude adjustment can be obtained by measuring the amplitude of the pilot carrier with a coherent detector.

Another problem that should be discussed is that of acquiring the desired signal, that is, causing each of the V. C. O.'s to lock to the correct frequency and phase. For a phase-locked loop to acquire a signal, the difference between the actual V. C. O. frequency and the correct V.C.O. frequency must be of the order of magnitude of locked-loop bandwidth or smaller. Acquisition is accomplished first by injecting into the first I.F. amplifier a signal strong enough to override other signals that may be present. This signal is tuned through the passband until all of the phase-lock loops have acquired it. It is then moved to the center frequency. This procedure aligns the frequencies of the V.C.O.'s. The first local oscillator is carefully tuned until the pilot carrier of the desired signal in each of the first I.F. amplifiers coincides with the injected signal in frequency. The injected signal is then removed and the pilot carrier of the desired signal is acquired by each of the loops.

The method of phase adjustment described here automatically compensates for any phase variations between channels up to the combining point. These variations include phase changes due to variations in the angle of arrival of the desired signal. motion of antenna elements, any drift in R.F. and I.F. amplifier phase shifts and phase variation in the cables connecting the preselectors and first mixers with the remainder of the receiving equipment.

The phase-locked array has an important disadvantage in that there is no way of distinguishing between signals of essentially the same frequency (within the loop bandwidth which may be on the order of a few cycles) and therefore it is susceptible to jamming.

Hopefully the presence of an interfering signal of essentially the same frequency as the desired signal will be rare, but if such an interfering signal is present a fixed common local oscillator with proper programmed phasing for each second mixer could be substituted for the V.C.O.'s. With this system the advantage of automatic compensation for phase shift in the part of each channel preceding the combining point is lost.

To summarize the above sections: It has been concluded that because of the effects of R.F. transmission line loss and the problems associated with R.F. delay lines, an H.F. array using I.F. combination is desirable. Also a useful method of phase adjustment using phase-locked loops to lock the phase of a pilot carrier to an I.F. reference oscillator has been described.

The chief advantage of a phase-locked system is its ability to not only correct for phase variations due to changes in the angle of arrival of the signal but in addition to correct for such system variations as could be caused by motion of the antenna elements, feed line variations and R.F. and I.F. amplifier instabilities.

The system described above is meant to convey the concept of a phase-locked array and should not be taken as the best possible configuration for such an array.

CHAPTER III
AN EXPERIMENTAL ARRAY

A six element experimental array has been constructed, along the lines of the hypothetical array described in the last section, to test the idea of automatic phasing. No attempt has yet been made to include provisions for automatic amplitude adjustment. The type and configuration of the antenna elements were chosen to make the signal and noise in all channels as uniform as possible so that elementary array theory could be used to predict the array pattern.

The experimental array, which is operated at a frequency of 30 mc, consists of six short stubs arranged in a linear array along the edge of the roof of the Antenna Laboratory Field Station. The stubs are spaced at 0.4 wavelength intervals giving a two-wavelength array. Extremely short stubs (approximately 0.05 wavelengths) were used in an attempt to reduce the effects of mutual coupling on the uniform amplitude distribution. In addition, 20 db pads were inserted at each element in order to reduce the effects of external interference so that each channel would have, essentially, an independent source of noise (the R.F. amplifier noise). These precautions were taken in order to provide more nearly ideal conditions for pattern measurement and to simplify the adjustment of channel gains. In practice, standard antenna elements would be used without pads.

Figure 3 shows one of the elements, with the housing for its associated R. F. amplifier, mounted on a railing at the edge of the roof. The entire array is shown in Fig. 4. The railing was covered with copper window screen in order to reduce the effects of the vertical members of the railing.

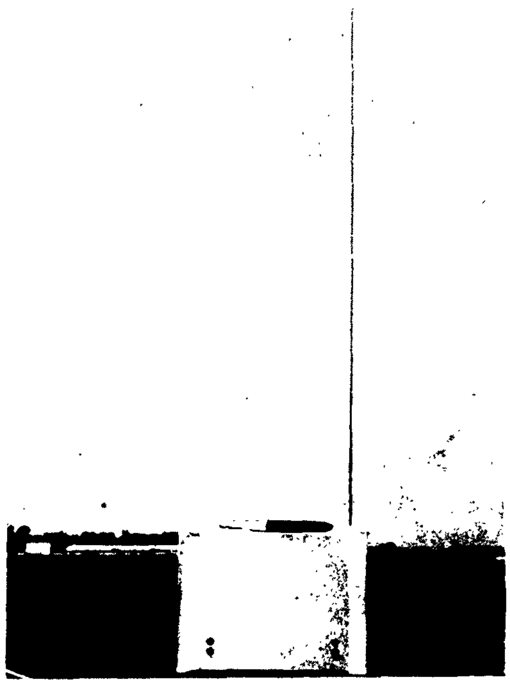


Fig. 3. Photograph of a single antenna element.

A block diagram of the array is shown in Fig. 5. The equipment is separated into two parts. The R. F. amplifier and first mixers are mounted with their respective elements. The remainder of the components are mounted in two rack cabinets as shown in Fig. 6.

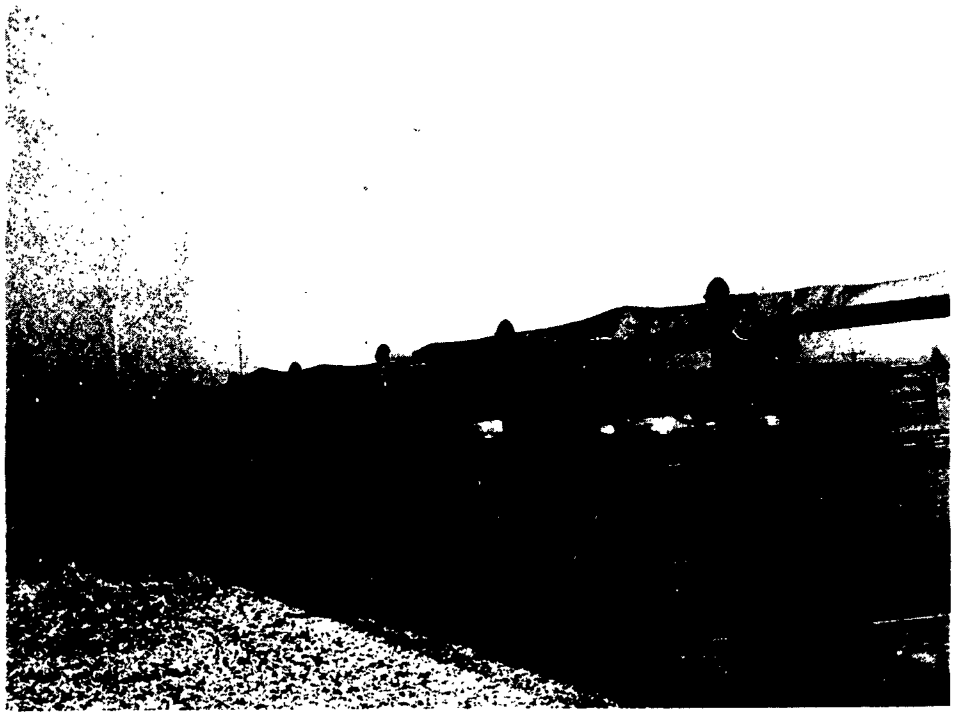


Fig. 4. Photograph of the entire array on the Antenna Laboratory Field Station roof.

All of the first mixers are fed from a common 44 mc local oscillator giving a first I.F. of 14 mc for each channel. Each of the R.F. amplifier-mixer units has a maximum gain of approximately 50 db. The gain is adjustable by a D.C. control voltage. Photographs of an R.F. amplifier-mixer unit removed from its housing are shown in Figs. 7 and 8. The circuit diagram is shown in Fig. 9.

No gain-control voltage was applied to the 6CB6 R.F. amplifier in order to insure uniform noise figure from unit to unit. The R.F. amplifier circuit gives a gain of about 35 db. A 6BE6 pentagrid

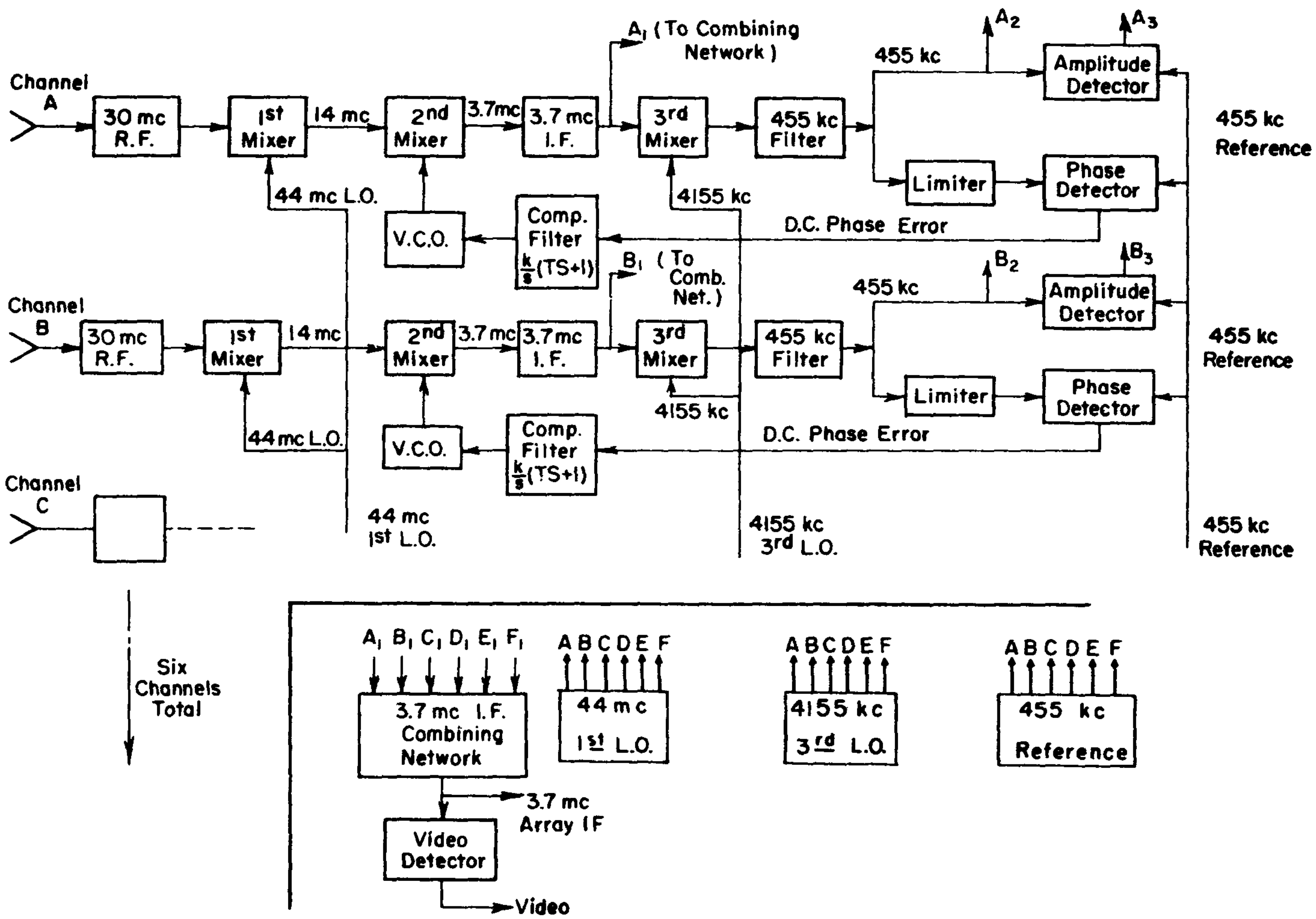


Fig. 5. Block diagram of the experimental array.

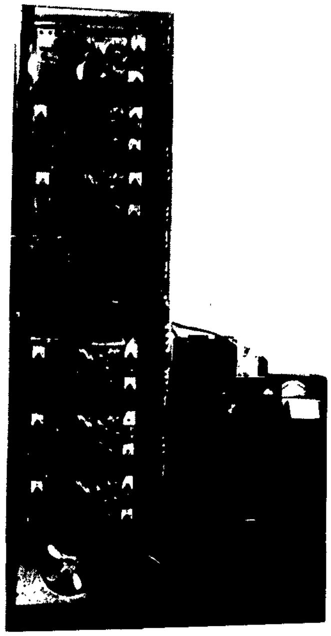


Fig. 6. The array receiving equipment. The phase-lock equipment is on the left and the signal measuring equipment and power supplies are on the right.

mixer was used for the mixer stage because its remote cut-off characteristic produces a smooth gain-control characteristic and to provide isolation, not attainable with a pentode mixer, of the signal and local oscillator inputs. This isolation is desirable since coupling of signal between R. F. units via the common local oscillator would affect the performance of the array.

A 51-ohm load was provided at the local oscillator input to provide a match for the long run of cable from the local oscillator. The somewhat unusual output network, which is actually a modified pi-network, was used to provide an exact match for the 14 mc I. F. signal cable.

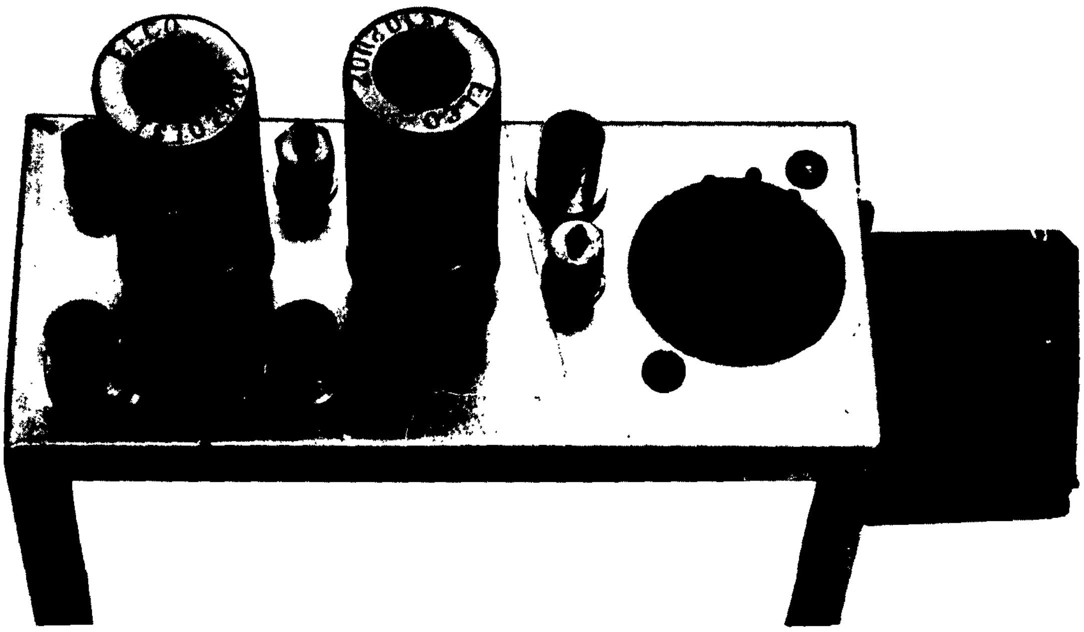


Fig. 7. Top view of the R.F. amplifier-first mixer unit.

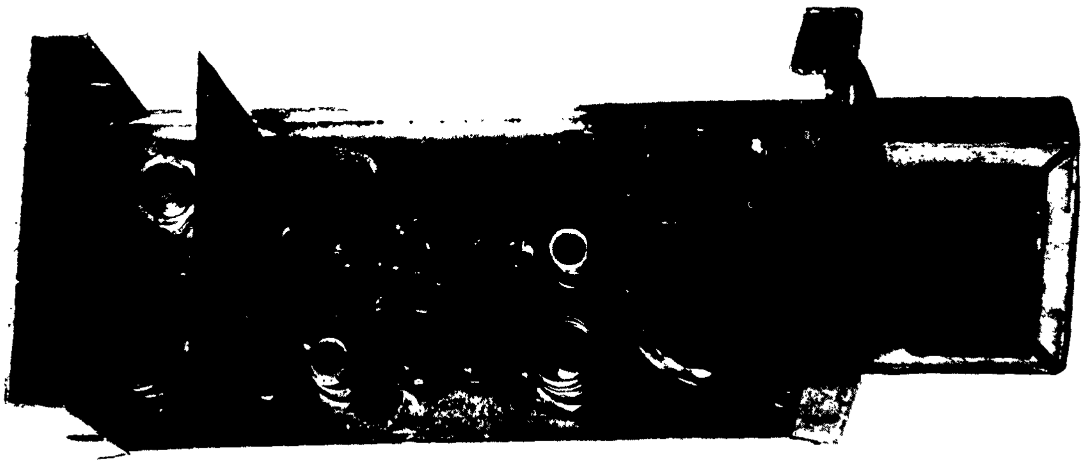
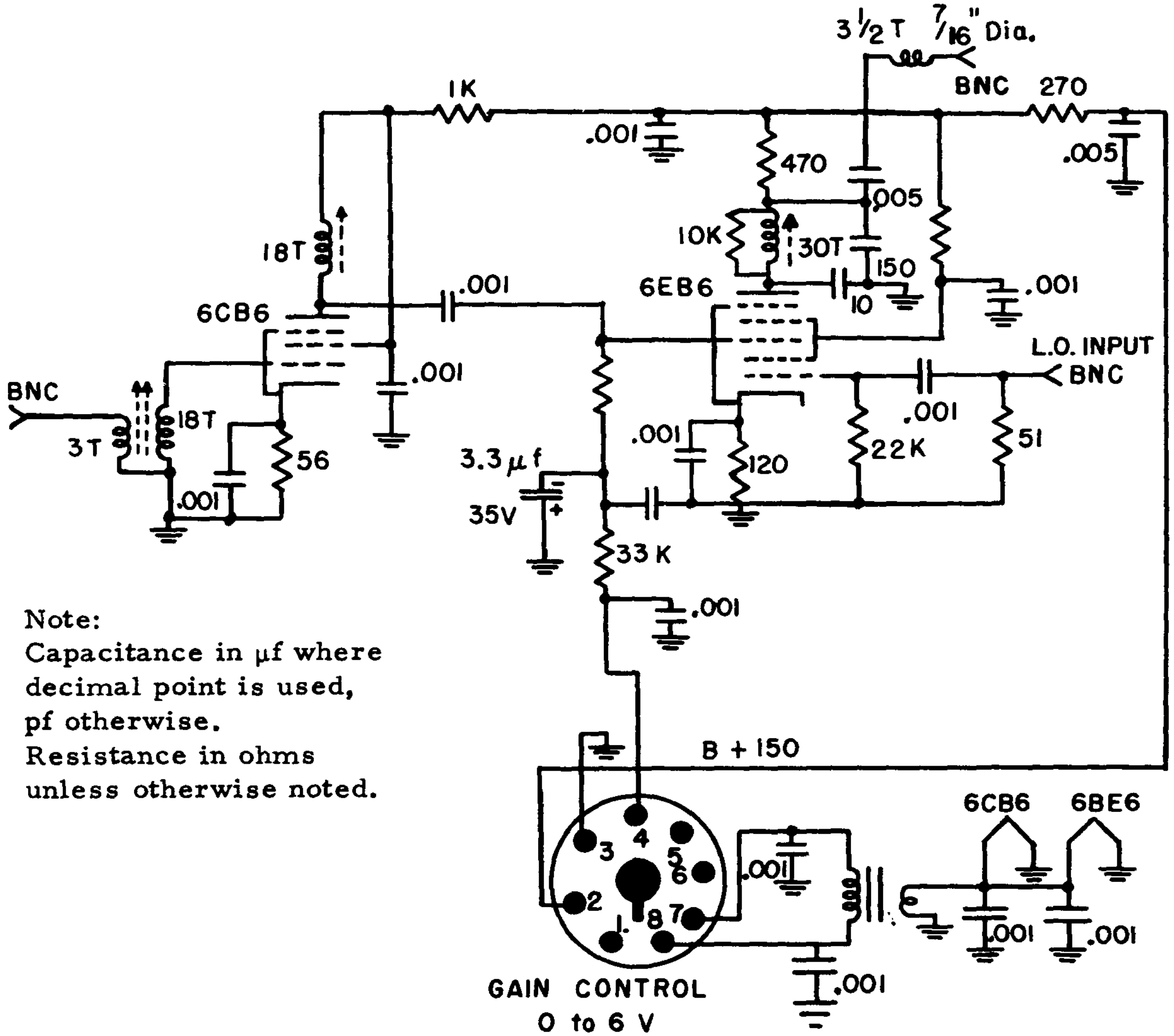


Fig. 8. Bottom view of the R.F. amplifier-first mixer unit.



Note:
Capacitance in μf where
decimal point is used,
pf otherwise.
Resistance in ohms
unless otherwise noted.

Fig. 9. Circuit diagram of the R.F. amplifier-first mixer unit.

The maximum gain of the mixer is approximately 15 db which with the 35 db gain of the R. F. amplifier gives an overall gain of 50 db. The overall noise figure is about 3 db. It was found to be independent of gain-control voltage. The variation in noise figure from unit to unit was found to be less than 0.05 db.

Referring to Fig. 5, the 14 mc I. F. is fed from each of the R. F. amplifier-mixer units on the roof through RG-58 cable to a set of units in the left-hand rack cabinet shown in Fig. 6, containing a second mixer, a voltage-controlled local oscillator, an active servo-compensating filter and a 3.7 mc second I. F. amplifier with a 100-kc bandwidth. The phase adjustment is provided by the second mixer. Figures 10 and 11 show photographs of one of the units. The circuit diagram is shown in Fig. 12.

A type 6AH6 pentode is used for the second mixer. The noise figure of this mixer was found to be important when the first mixer was operated at reduced gain. To improve the noise figure it was found necessary to have the first mixer output load the second mixer input heavily. In other words, the second mixer was provided with a high (around 2K ohm) input impedance. The noise figures of the second mixers varied between 10 and 13 db.

An International Crystal type FO-11 crystal oscillator, the frequency of which is pulled by a 1N951 voltage-variable capacitor,

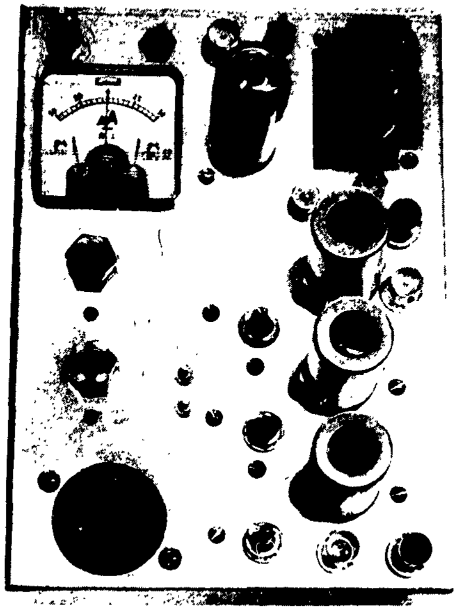


Fig. 10. Top view of the second mixer-V.C.O.-3.7 mc I.F. chassis.

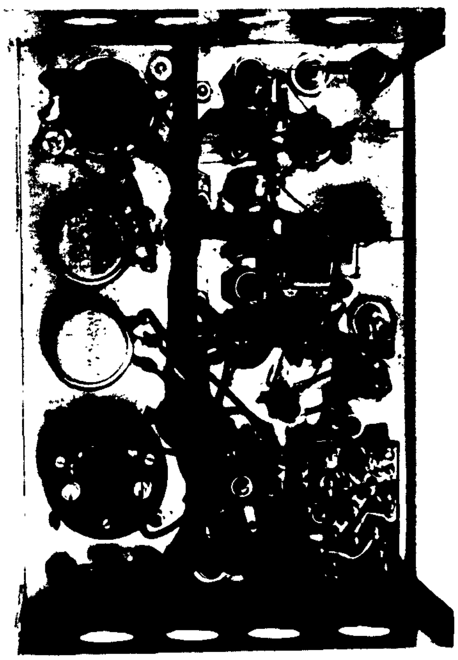
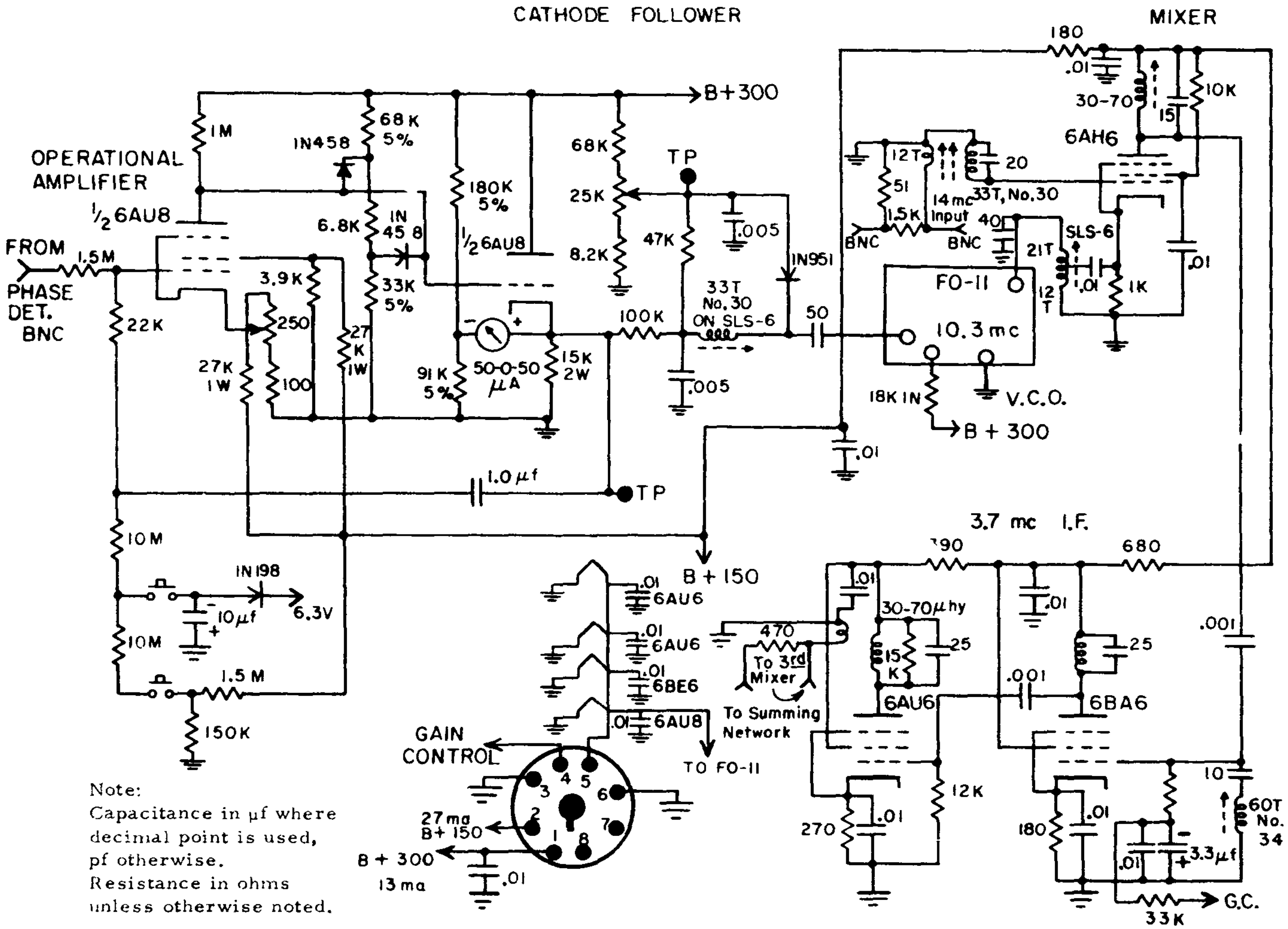


Fig. 11. Bottom view of the second mixer-V.C.O.-37 mc I.F. chassis.



Note:
Capacitance in μf where decimal point is used, pf otherwise.
Resistance in ohms unless otherwise noted.

Fig. 12. Circuit diagram of the second mixer-V.C.O.-3.7 mc I.F. chassis.

is used for a V.C.O. The control voltage is fed to the 1N951 diode through a 10 μhy slug-tuned coil. The coil is approximately resonant with the capacitance of the 1N951. Adjusting the coil tunes the center frequency of the crystal oscillator. The bias provided by the 25K pot at the anode end of the 1N951 diode adjusts both the center frequency and the voltage sensitivity since the diode has a non-linear voltage-capacitance characteristic. The sensitivity of the V.C.O. is on the order of 1000 radians per second per volt.

The control voltage is obtained from the output of an operational amplifier consisting of a type 6AU8. The pentode half of the 6AU8 is used as a high gain D.C. amplifier having a gain of about 400 while the triode section is used as a cathode follower to provide a low output impedance of about 150 ohms. The meter in the cathode circuit indicates frequency. Feedback through the 1.0 μf capacitor and 22 K resistor provide the required proportional-plus-integral compensation.

The calculated transfer function of the compensator is given by

$$G = .66 \frac{.02\, s + 1}{s} \tag{2}$$

where G is the transfer function and s is the Laplace transform variable. Push-button tuning is provided by injecting a small positive or negative current at the junction of the 1.0 μf capacitor and the 22K resistor. The push buttons are used to acquire the signal.

The 3.7 mc I.F. amplifier circuit is fairly straight-forward. A 6BA6 is used as a gain-controlled stage and is followed by a 6AU6 fixed-gain stage. Any phase shift due to a change in gain-control voltage is unimportant at this point since it comes before the 3.7 mc summing point. Part of the 3.7 mc output is fed to a chassis containing a third mixer and amplitude and phase detectors, and part is sampled through a 470-ohm resistor and fed to the summing network.

Photographs of the phase-detector chassis appear in Figs. 13 and 14, and the circuit diagram is shown in Fig. 15. For two reasons the signal is first converted to 455 kc by the 6BE6 mixer, using a 4155 kc local oscillator which is common to all the mixers. One reason for conversion to 455 kc is the availability of 455 kc transformers. The other reason is the necessity of a narrower bandwidth preceding the phase detector. The narrow bandwidth is obtained by the use of two Miller 12-cl 455 kc I.F. transformers critically coupled in cascade. This provides an approximately rectangular passband of 10 kc width at the 6 db points. While 10 kc is certainly not narrow when compared with bandwidths of a few hundred cycles which may be necessary in practice, it is narrow when compared

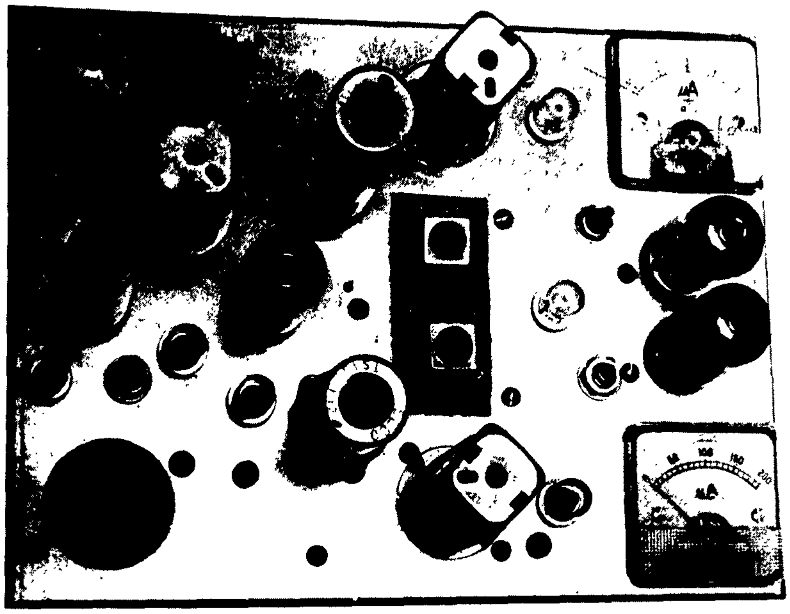


Fig. 13. Top view of the phase detector chassis.

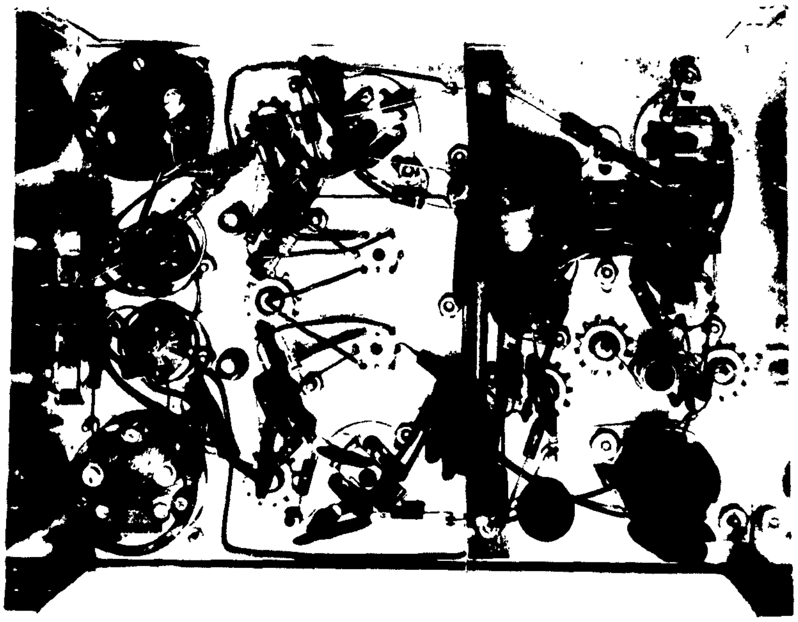
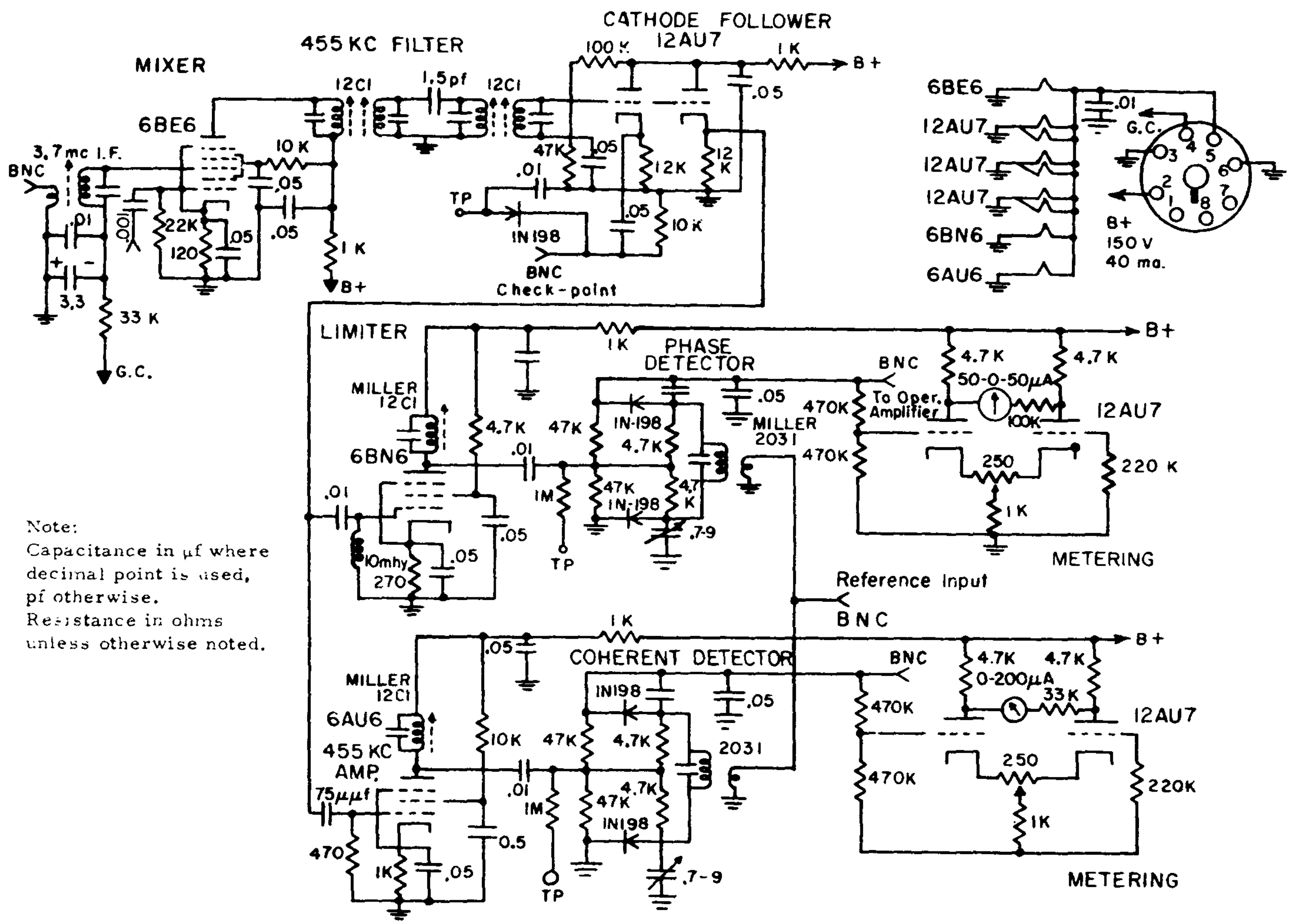


Fig. 14. Bottom view of the phase detector chassis.



Note:
Capacitance in μf where decimal point is used, pf otherwise.
Resistance in ohms unless otherwise noted.

Fig. 15. Circuit diagram of the phase detector chassis.

with the information bandwidth which can be taken to be the 100-kc bandwidth of the 3.7 mc second I.F. amplifier.

The output of the 455-kc filter is fed to two cathode followers consisting of the two halves of a 12AU7. One cathode follower provides a point at which the signal may be monitored and drives a peak detector which is used as a test point for tuning and gain adjustment. The output of the other cathode follower is fed to the phase detector and a coherent amplitude detector.

The phase detector consists of a 6BN6 gated beam tube used as a limiter followed by a two-diode balanced mixer which is used to compare the output phase of the 6BN6 limiter with the phase of the common 455-kc reference oscillator. The output is fed back to the compensator on the V.C.O. chassis. It is also metered by the 12AU7 metering circuit. The sensitivity of the phase detector is 6.4 volts per radian. When this sensitivity is considered along with that of the V.C.O. and the transfer function of the compensator is taken into account, a closed-loop bandwidth of about 10 cycles results.

The coherent amplitude detector consists of a 90° phase-shift network, a 6AU6 I.F. amplifier-driver and a balanced mixer identical to the one used for the phase detector. The phase-shift network consists of the 75 pf capacitor and the 470-ohm resistor at the grid of the 6AU6 amplifier. If the phase detector is considered to measure the sine of the phase difference, due to the 90° phase shift the output of the amplitude detector will be proportional to the amplitude of the signal and the cosine of the phase. This is the result that would be obtained if the amplitude detector were considered to multiply the signal by the reference. Output of the amplitude detector is provided at a BNC fitting and is also monitored by a 12AU7 metering circuit.

The circuit of the combining network is shown in Fig. 16.

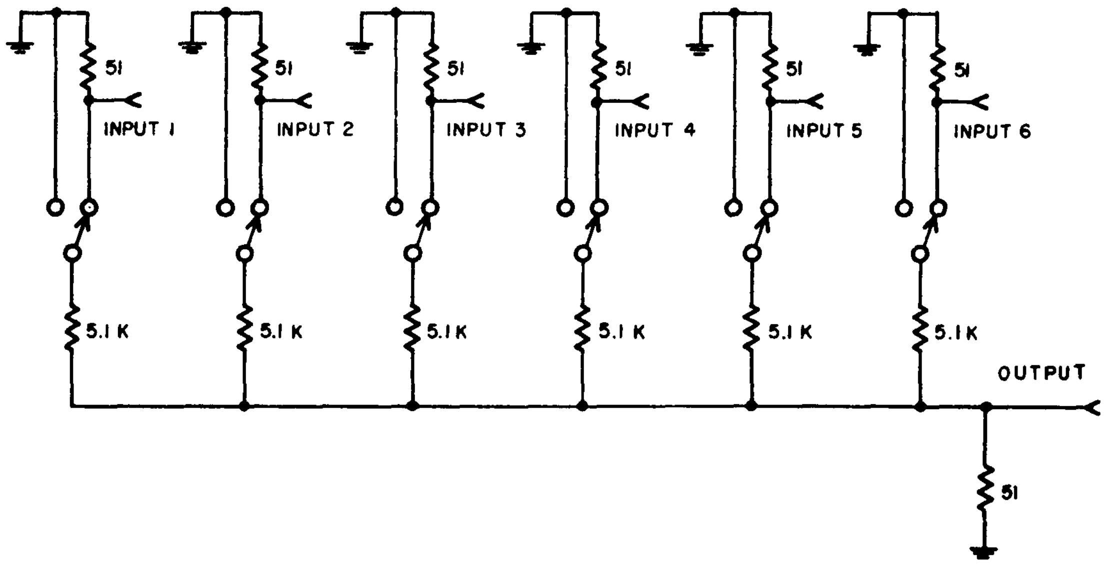


Fig. 16. Circuit diagram of the combining network.

A 51-ohm resistor is used as a summing resistor. The attenuation between each of the 3.7 mc I.F. amplifiers and the summing point is 60 db, so that the channels are sufficiently well isolated to make interaction negligible. If the signals at the outputs of the 3.7 mc I.F.

amplifiers are in phase, the signals arriving at the summing point should be in phase. The difference in path length is small enough to be neglected at 3.7 mc. The switches shown in the circuit switch each of the channels in or out of the summing network without affecting any of the other channels.

The output of the summing network is at too low a level for the phase to be measured directly by an oscilloscope, so another technique must be used to insure proper phasing. A 3.7 mc signal is first fed to the input of each of the third mixers and the tuned circuits are carefully aligned. This provides a minimum amount of phase difference from channel to channel between the mixer input, which is the point where the signals are sampled for summing, and the phase detectors. The 3.7 mc signal generator is then detuned approximately 20 cycles so that the third I.F. frequency differs from the 455 kc reference by 20 cycles. The output of each of the phase detectors will then be a 20 cycle sine wave.

The phasing of the phase detector outputs are observed in pairs by a dual-trace oscilloscope. Any small phase differences are corrected by slightly detuning the plate circuit of the 6BN6 limiters. This insures that when the loop is closed and the phase detector outputs are zero, the signals at the inputs to the third mixers are in phase.

The summing network panel is shown in the lower half of Fig. 17. The knobs control the switches which switch each of the channels in

Fig. 17. Photograph showing the summing network panel (below) and the various oscillator chassis (above).

and out of the network. The screw-driver adjustments below each knob are the gain-control voltage adjustments. The chassis in the upper half of Fig. 17 contain the 44-mc local oscillator, the 4155-kc local oscillator and the 455-kc reference oscillator. The oscillators were purchased as assembled units from International Crystal Manufacturing Co. The circuit diagrams of the oscillators and their associated amplifiers are given in Appendix C and should be self-explanatory. The circuit of the International Crystal type FO-11 oscillators used as V.C.O.'s is also given in Appendix C.

CHAPTER IV
PATTERN MEASUREMENT

As mentioned previously, an indication of the ability of a phase-locked array to reject unwanted signals can be obtained from the array pattern. Although a phase-locked array has a pattern in the normal sense, it can't be measured by conventional methods since the beam automatically follows the signal source giving a constant output as a function of angle. To obtain a meaningful indication of array performance, a signal source must be set up at a fixed angle and an interference rejection pattern measured as an interfering signal source is moved around the array. Accordingly, interference rejection patterns were measured for three angles of the desired signal for comparison with the theoretical patterns for these angles.

The measurements were performed in the following manner. The output of the combining network of the array was connected to the input of a Collins 75S-3 receiver. The receiver was tuned to 3680 kc which is the frequency produced at the output of the 3.7 mc summing network by a 30.02 mc interfering signal source when the array is locked to a 30 mc desired signal source (the circuit is given in Appendix C). This 3680 kc signal falls well within the passband of the 3.7 mc I.F. The 440 kc produced by the third mixer, however, is about 30 db down in the phase-detector pass-band and therefore can be neglected as far as any effects on automatic phase adjustment are concerned. The receiver is operated with the A. V. C. off and the product detector is used. The product detector converts the signal directly to video and produces an audio frequency tone at the audio output. This output is filtered by an audio filter with a 20 cycle bandwidth to remove any noise that may be present and then passed through a step attenuator. The output of the attenuator is fed to a power-indicating device which consists of a temperature-limited diode (type 5722) the plate current of which is measured by a microammeter. The meter has a range of about 1 db and the attenuator is variable in 1 db steps up to 100 db. The arrangement used for measuring undesired signal strength is shown in Fig. 18.

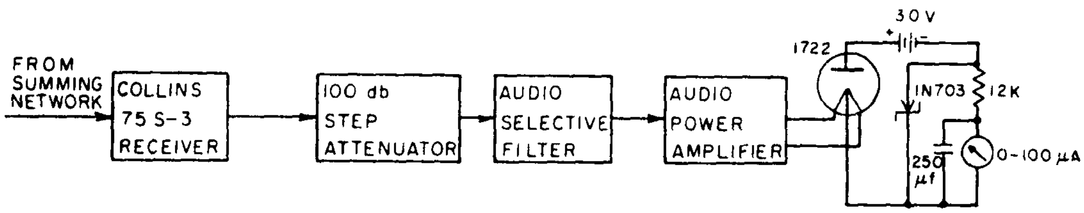


Fig. 18. Set-up used for signal strength measurements.

The measurement procedure used is as follows. The audio filter is first switched out of the circuit so that the noise power output of the receiver can be measured. The gain of each channel is then carefully adjusted to provide equal noise powers in each channel. This insures essentially equal gain since the noise figures of each of the channels are almost identical. The desired signal source is then placed at some fixed angle with respect to the center of the array and its amplitude is adjusted to give a signal-to-noise ratio of about 10 db in the phase-detector bandwidth. For the present array this produces a 17 db signal-to-noise ratio for each channel at the receiver output since the receiver bandwidth is one fifth the phase detector bandwidth. The amplitude of the undesired signal is set to be about the same as that of the desired signal. Each of the channels is then switched into the summing network and the pattern of the undesired signal is measured as a function of angle, as the undesired signal source is moved in a circular arc around the center of the array. Figure 19 is a sketch of the array showing surrounding objects. The angles of the desired and undesired signal sources are also defined.

In practice it turned out that equal signal amplitudes at all elements were impossible to achieve. From broadside ($\theta = 90°$) to about 65° the amplitudes were uniform within ±2 db but at angles greated than 105 degrees, where the trees and power lines probably scattered some signal, the amplitudes of the signals at some of the elements varied as much as 8 db from the average. At angles

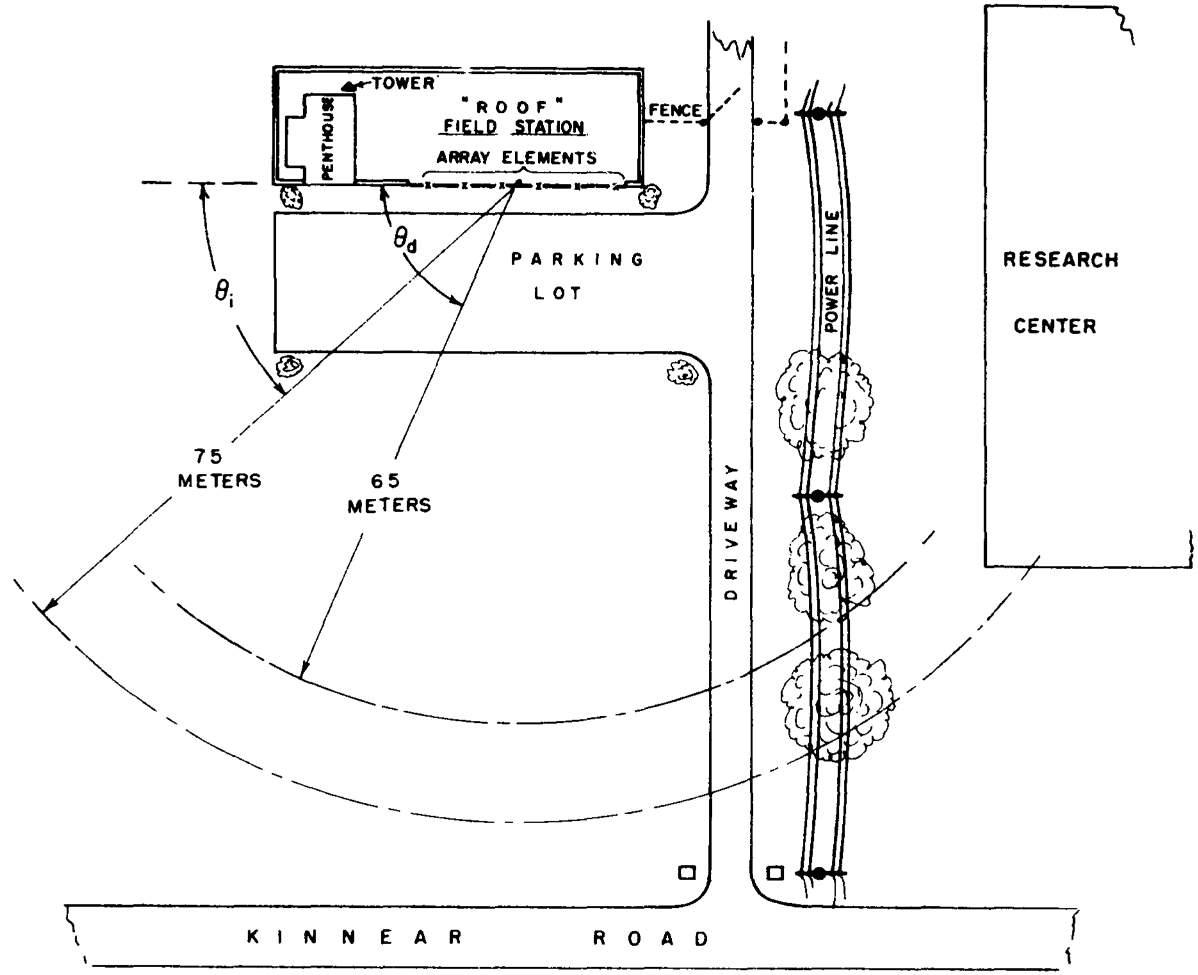


Fig. 19. Sketch showing location of the array and its surroundings. The angles of the desired (θ_o) and undesired (θ_i) signals are shown.

below 65° the amplitudes had a gradual increasing trend as the angle decreased. This was possibly due to the effects of the penthouse and the tower near the array.

The average amplitude was measured as a function of angle by moving the 30-mc desired signal source along the arc used for pattern measurements with all of the channels switched into the summing network. Figure 20 shows the average amplitude plotted

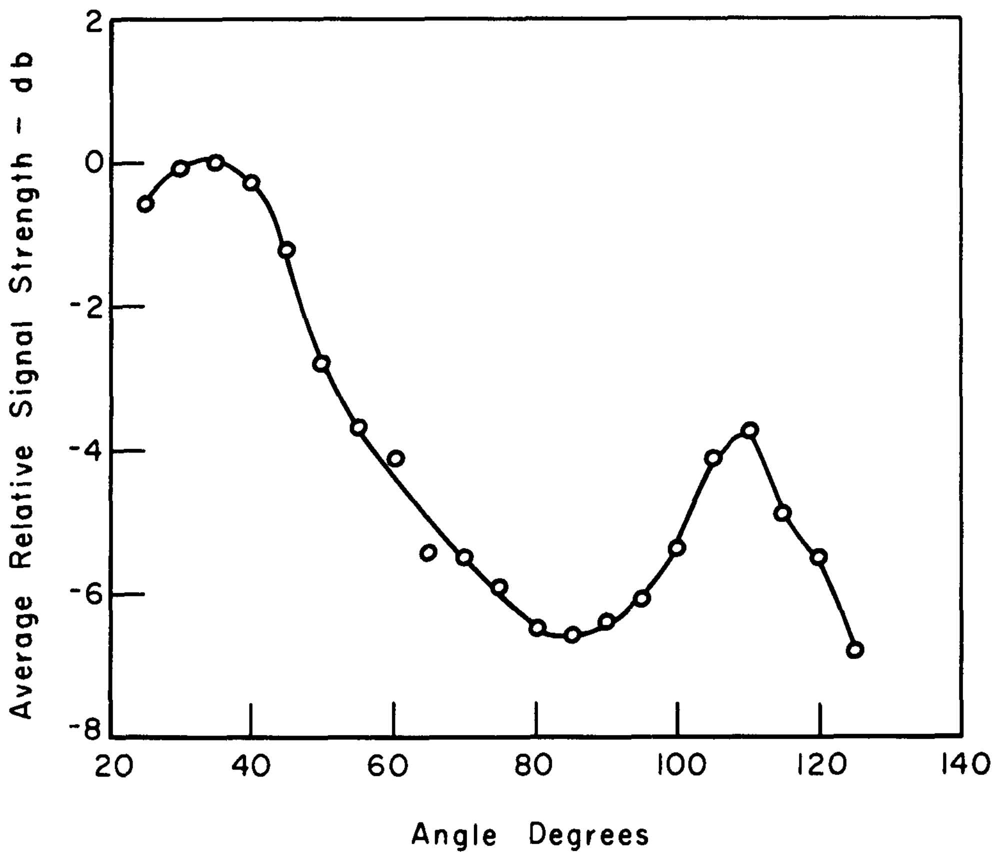


Fig. 20. Curve of the average amplitude over the array as a function of angle.

as a function of angle at the 5° intervals used for pattern measurements.

Figures 21, 22, and 23 show theoretical patterns and measured points for three angles of the desired source, 60°, 75°, and 90°, respectively. The theoretical patterns were calculated by assuming uniform amplitudes and with phasing such as to cause the desired signals in each channel to combine in phase. Thus the patterns are the same as the patterns of a conventional, uniform amplitude, phased array. The measured points have been corrected to take into account the variation in average amplitude as a function of angle.

The agreement between the calculated patterns and measured points is surprisingly good considering the nonuniformity in signal amplitude from element to element.

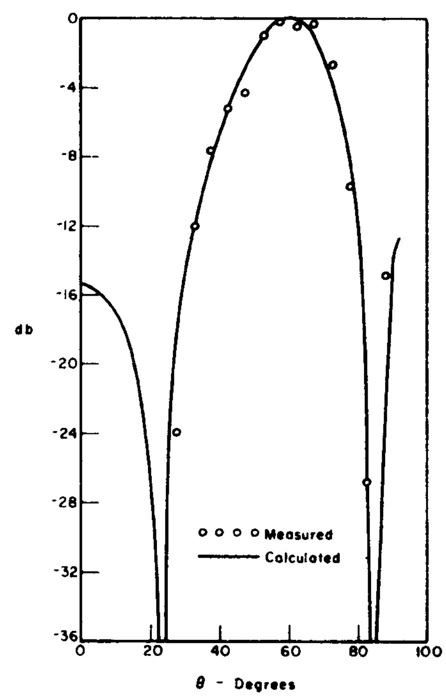


Fig. 21. Calculated (solid curve) interference rejection pattern and measured points (small circles) for the angle of the desired signal $\theta_d = 60^o$.

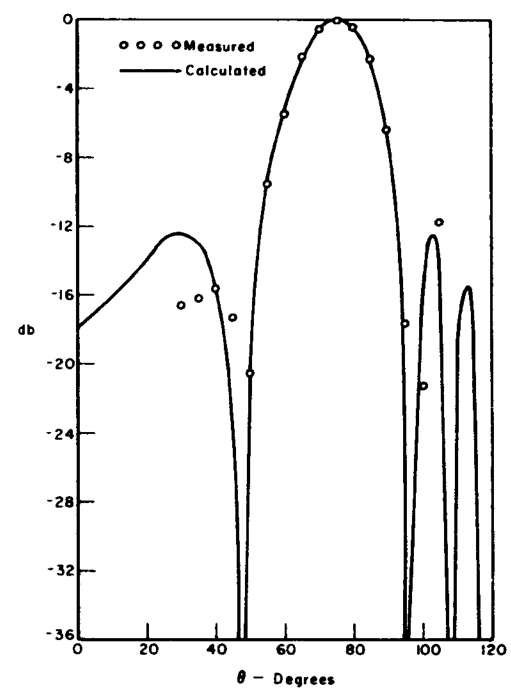


Fig. 22. Calculated (solid curve) interference rejection pattern and measured points (small circles) for the angle of the desired signal $\theta_d = 75^o$.

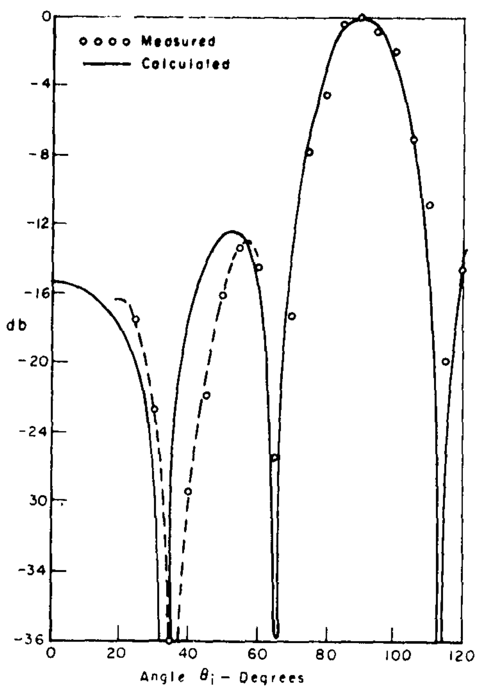


Fig. 23. Calculated (solid curve) interference rejection pattern and measured points (small circles) for the angle of the desired signal $\theta_d = 90^{\circ}$.

CHAPTER V
CONCLUSIONS AND RECOMMENDATIONS

It has been shown that a phase-locked array can, under most circumstances, give interference rejection comparable to that of a conventionally phased array. In addition it has the advantage of automatically tracking the desired signal and correcting for phase variations due to antenna motion, mutual coupling between elements, scattering from near field obstructions, and instabilities in the signal channels between the antenna elements and the signal-combining point.

A disadvantage of the phase-locked array is that its operation is affected by the presence of interference and by noise in the phase detector. It has been shown that an interfering signal within the phase-detector pass-band causes modulation of the desired signal with a corresponding reduction of gain (see Appendix B).

Further theoretical and experimental studies should be conducted. The theoretical work should include development of methods for predicting the performance of the array in the presence of both independent and coherent noise. The case of coherent noise is especially important since it could represent the case where a sideband of an interfering signal falls in the phase-detector passband.

This would be far more common than having a carrier in the pass-band.

To check theoretical results consideration should be given to modeling the experimental array at U. H. F. or the lower microwave region. This would make it possible to make measurements under much more ideal conditions.

The use of a phase-locked array will provide more reliable communications than a conventional array because of its automatic tracking capability. In addition, most of the existing components of a phase-locked array can be used without modification if it is desired to use the array in a more conventional programmed-phase mode of operation.

APPENDIX A

OPTIMIZING SIGNAL-TO-NOISE RATIO FOR A TWO ELEMENT ARRAY WITH INDEPENDENT NOISE

As a starting point in the optimization problem a simple array of two elements will be considered, using the assumption of independent noise in each channel. The noise power in each channel will add directly at the combining point. That is, if the noise power referred to the input of the first channel is N_1 and the noise power referred to the input of the second channel is N_2 and the channel voltage gains are a_1 and a_2, respectively, the total noise power at the output of the combining network will be

(A-1) $$N_O = a_1^2 N_1 + a_2^2 N_2 \ .$$

This result is independent of the phasing adjustment in each channel. Ovbiously optimum phasing occurs when the signals are combined in phase, since this results in the largest signal voltage and the phase adjustment has no effect on the combined noise power. Assuming the signals have been adjusted to be in phase, the optimum values for a_1 and a_2 can be determined. Let A_1 be the signal amplitude referred to the input of the first channel and A_2 the signal amplitude referred to the input of the second channel then the signal power S_O is given by

(A-2) $$S_o = [a_1 A_1 + a_2 A_2]^2 .$$

The signal-to-noise ratio is obtained by dividing (2) by (1) giving

(A-3) $$\frac{S_o}{N_o} = \frac{[a_1 A_1 + a_2 A_2]^2}{a_1^2 N_1 + a_2^2 N_2} .$$

Substituting γ for $\frac{a_2}{a_1}$ gives

(A-4) $$\frac{S_o}{N_o} = \frac{A_2}{N_2} \frac{\left[\gamma + \frac{A_1}{A_2}\right]^2}{\gamma^2 + \frac{N_1}{N_2}} .$$

The quantity $\frac{S_o}{N_o}$ may be maximized by taking a derivitive with respect to γ and setting the result equal to zero. Differentiation gives

(A-5) $$\frac{d}{d\gamma}\left(\frac{S_o}{N_o}\right) = \frac{-\frac{A_1}{A_2}\gamma^2 + \left[\frac{N_1}{N_2} - \left(\frac{A_1}{A_2}\right)^2\right]\gamma + \frac{N_1}{N_2}\frac{A_1}{A_2}}{\left[\gamma^2 + \frac{N_1}{N_2}\right]^2} .$$

Setting the numerator equal to zero and solving for γ

(A-6) $$\gamma = \frac{1}{2}\left[\frac{A_2 N_1}{A_1 N_2} - \frac{A_2}{A_1}\right] \pm \frac{1}{2}\left[\frac{A_2 N_1}{A_1 N_2} + \frac{A_2}{A_1}\right] .$$

Using the negative sign in Eq. (A-6) obviously gives the wrong result since in this case the signals would cancel. The positive sign gives the desired maximum

$$\text{(A-7)} \qquad \frac{a_2}{a_1} = \gamma = \frac{A_2}{A_1} \frac{N_1}{N_2} \quad .$$

The relative gains of the two channels should be proportional to the amplitudes of the signals and inversely proportional to the relative noise power in each channel. The phase in each channel can be optimized independently of the amplitude.

APPENDIX B

EFFECT OF INTERFERING MONOCHROMATIC SIGNAL ON PERFORMANCE OF PHASE-LOCKED ARRAY

In order to determine the effects of a single sinusoidal interfering signal on the performance of a phase-locked array the effect of the signal on a single phase-locked loop must be determined.

In the following analysis the subscript d will be used to refer to the desired signal while o refers to the V. C. O. output. A list of symbols used is given at the end of the section.

Referring to Fig. B-1 the input signal to the mixer is given by

$$v_{in} = A_d \cos(\omega_d t + \phi_d) + A_i \cos[(\omega_d + \omega_i)t + \phi_i] \tag{B-1}$$

where ω_i is the frequency difference between the desired and undesired signals. v_{in} may be represented by the expression

$$v_{in} = A_{in}(t)[\cos \omega_d t + \phi_{in}(t)] \tag{B-2}$$

which is an amplitude and phase modulated signal with carrier frequency ω_d. When the interfering signal amplitude A_i is zero, the phase angle $\phi_{in}(t)$ will be equal to ϕ_d, the phase angle of the desired signal. As A_i increases, $\phi_{in}(t)$ will take on some periodic variation with time.

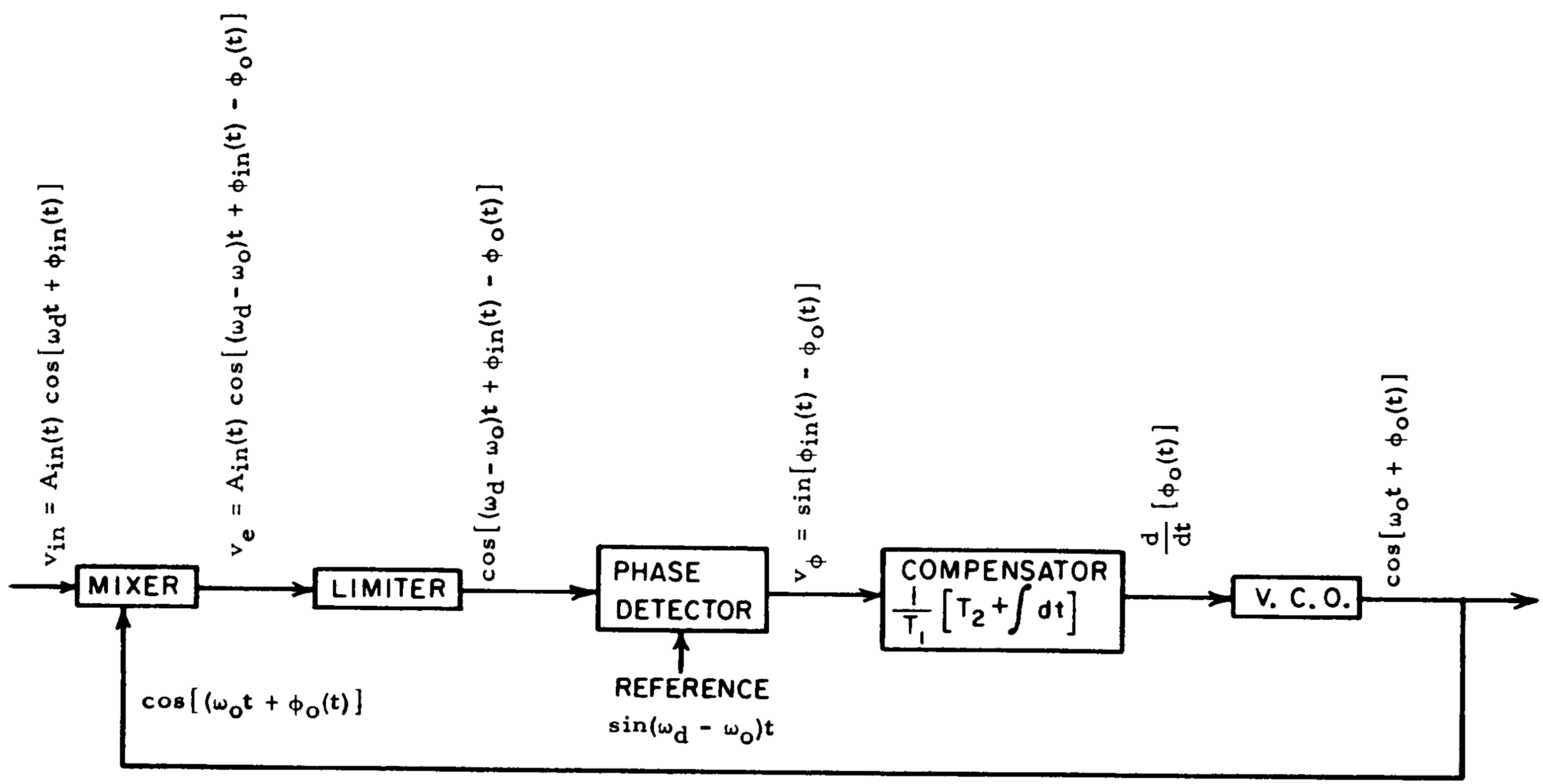


Fig. B-1. Phase-locked loop.

With v_{in} applied to the mixer, its output will be given by

(B-3) $$v_e = A_{in}(t) \cos[(\omega_d - \omega_o)t + \phi_{in}(t) - \phi_o(t)] .$$

The amplitude variations are removed by the limiter and the sine of the difference between ϕ_{in} and ϕ_o is measured by the phase detector and used to control the V.C.O. If the variations in ϕ_{in} are very slow corresponding to small ω_i, ϕ_o follows these variations accurately. The error is small and the sine function can be linearized. The loop, however, is unable to follow rapid variations of ϕ_{in}. This is due in part to the low pass filter action of the compensator and the V.C.O. The variations in output phase are further reduced by the fact that the output of the phase detector is limited to unity. Because of the relatively small amplitude of the variations in ϕ_o these variations should have little effect on the output spectrum of the phase detector. In fact, opening the loop should make little difference to the A.C. variation of ϕ_o.

An open loop representation for determination of the A.C. variations in ϕ_o is given in Fig. B-2. The fixed local oscillator phase is taken to be equal to ϕ_d. The mixer output v_e' is then given by

(B-4) $$v_e' = A_d \cos(\omega_d - \omega_o)t + A_i \cos[(\omega_d - \omega_o)t + \omega_i t + \phi_i - \phi_d] .$$

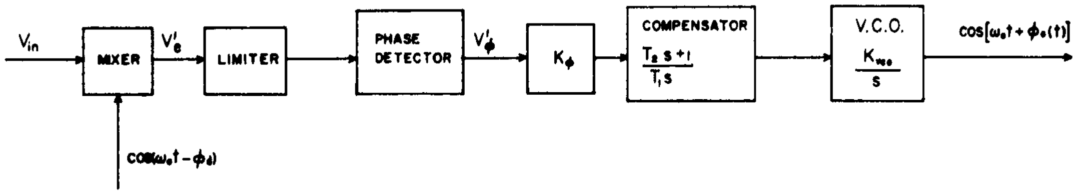


Fig. B-2. Open loop representation of the phase-locked loop.

Rewriting Eq. (B-4) gives

$$\text{(B-5)} \qquad v_e' = [A_d + A_i \cos(\omega_i t + \phi_i - \phi_d)]\cos(\omega_d - \omega_o)t$$
$$-A_i \sin(\omega_i t + \phi_i - \phi_d) \sin(\omega_d - \omega_o)t \;.$$

This may be represented by an amplitude and phase modulated carrier.

$$\text{(B-6)} \qquad v_e' = A_e'(t) \cos[(\omega_d - \omega_o)t + \phi_e'(t)] \;.$$

The output of the phase detector is given by

$$\text{(B-7)} \qquad v_\phi' = \sin \phi_e' = \frac{\sin(\omega_i t + \phi_i - \phi d)}{[s^2 + 1 + 2s \cos(\omega_i t + \phi_i - \phi_d)]^{\frac{1}{2}}}$$

where

$$s = \frac{A_d}{A_i} \quad .$$

This is an odd periodic function and can be represented by a Fourier series of sine functions

(B-8) $$v'_\phi = a_1 \sin(\omega_i t + \phi_i - \phi_d) + a_2 \sin 2(\omega_i t + \phi_i - \phi_d) + \cdots .$$

The first coefficient a_1 will be calculated to determine the effect of the first term on the output phase. The effects of the higher terms will be much smaller due to the low pass filter action of the compensator and V.C.O. and therefore they will not be considered. The first coefficient a_1 is given by

(B-9) $$a_1 = \frac{\omega_i}{\pi} \int_{\frac{-\pi}{\omega_i}}^{\frac{\pi}{\omega_i}} v'_\phi(t) \sin(\omega_i t + \phi_i - \phi_d) dt$$

$$= \frac{1}{\pi} \int_{-\pi}^{\pi} \frac{\sin^2 x \, dx}{[s^2 + 1 + 2s \cos x]^{\frac{1}{2}}} .$$

Let

$$s' = \frac{2s}{s^2 + 1} .$$

Equation (B-9) becomes

(B-10) $$a_1 = \frac{1}{\pi \sqrt{s^2 + 1}} \int_{-\pi}^{\pi} \frac{\sin^2 x \, dx}{[1 + s' \cos x]^{\frac{1}{2}}} .$$

The denominator of the integrand in (B-10) can be expanded in a binomial series

$$\text{(B-11)} \qquad \frac{1}{[1 + s' \cos x)]^{\frac{1}{2}}} = b_o - b_1 \, s' \cos x + b_2 \, s'^2 \cos^2 x - b_3 \, s'^3 \cos^3 x + \cdots .$$

The coefficients are given by

$$\text{(B-12)} \qquad [b_n] = 1, \frac{1}{2}, \frac{1}{2}\cdot\frac{3}{4}, \frac{1}{2}\cdot\frac{3}{4}\cdot\frac{5}{6}, \ldots, \frac{1}{2}\cdot\frac{3}{4}\cdot\frac{5}{6}\cdots\frac{2n-1}{2n}, \cdots .$$

The coefficient a_1 is then obtained by substituting the series (B-11) into Eq. (B-10) giving

$$\text{(B-13)} \qquad a_1 = \frac{1}{\pi\sqrt{s^2+1}} \int_{-\pi}^{\pi} [b_o \sin^2 x - b_1 \, s' \sin^2 x \cos x + b_2 \, s'^2 \sin^2 x \cos^2 x + \cdots]$$

which may be integrated term by term. The value of the integral

$$\text{(B-14)} \qquad I_{2n} = \int_{-\pi}^{\pi} \cos^{2n} x \sin^2 x \, dx$$

can be found by substitution into standard forms to be

$$\text{(B-15)} \qquad I_{2n} = \frac{\pi}{n+1} b_n$$

where the b_n are the coefficients of (B-12). By substitution into (B-13) a_1 is found to be given by

(B-16) $$a_1 = \frac{1}{\sqrt{s^2 + 1}} \sum_{n=0}^{\infty} \frac{b_{2n} b_n}{n+1} s^{\prime 2n} \quad .$$

The first six terms of the series were evaluated and the results are shown in Fig. (B-3) as a function of s.

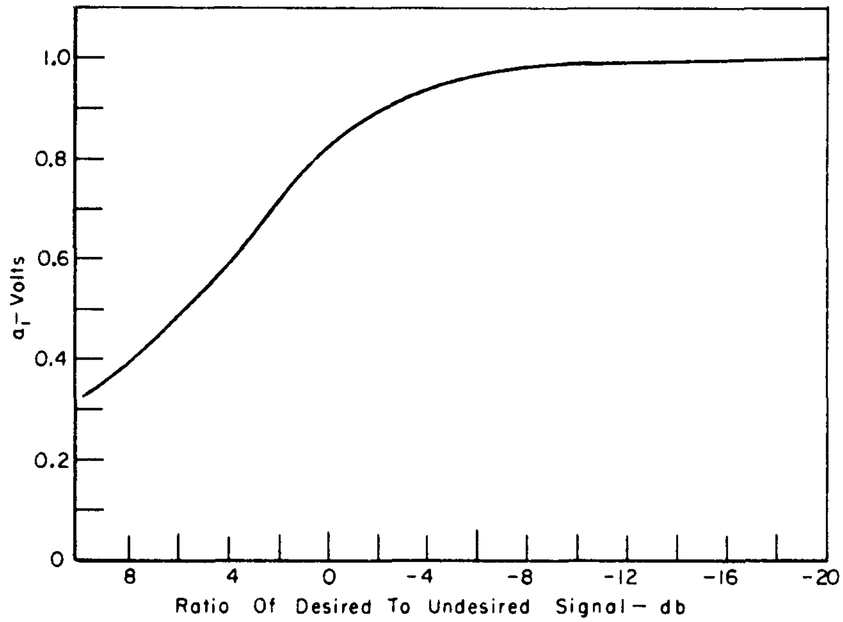


Fig. B-3. Curve showing the first Fourier coefficient, a_1, of the phase detector output as a function of s, the ratio of desired to undesired signal.

The amount of phase modulation in the V. C. O. output due to the fundamental term in the phase detector output can be determined by simply multiplying the magnitude of the complex transfer function of the compensator and V. C.O . by a_1. The additional phase shift introduced can be ignored for the case where all channels are identical since each will give the same phase shift. In the experimental array the transfer function $F(j\omega_i)$ of the compensator and V.C.O. is

$$(B\text{-}17) \qquad F(j\omega_i) = 4300\left[\frac{j\,0.02\omega_i + 1}{-\omega_i^2}\right] .$$

The magnitude of $F(j\omega)$ is given by

$$(B\text{-}18) \qquad |F(j\omega_i)| = 4300\,\frac{[0.0004\,\omega_i^2 + 1]^{\frac{1}{2}}}{\omega_i^2} .$$

The amplitude of the fundamental component in ϕ_o is shown as a function of s for several values $f_i = \frac{2\pi}{\omega_i}$ in Fig. B-4.

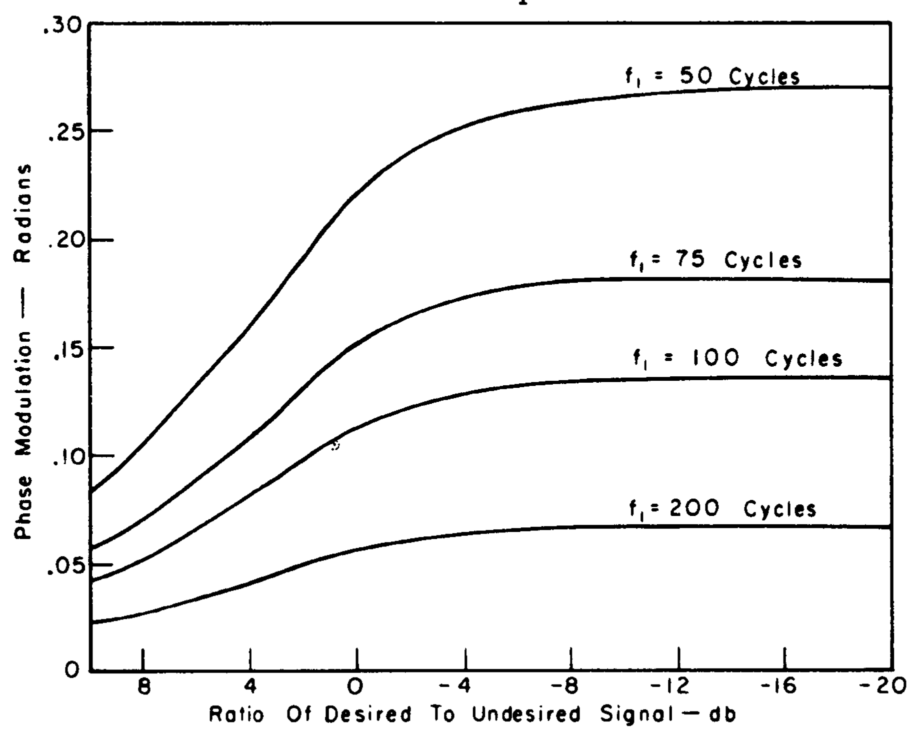


Fig. B-4. Curve showing the first Fourier coefficient of the V.C.O. phase modulation as a function of s, the ratio of desired to undesired signals, for several difference frequencies.

The effect of an interfering signal on the performance of a linear array can be determined by letting the signal in each channel be phase modulated by the V.C.O. output and then adding the signals. The signal from the n^{th} channel may be represented using complex notation as

(B-19) $$V_{dn} = e^{j\phi_n}$$

where ϕ_n is the phase modulation in the n^{th} channel and is given by

(B-20) $$\phi_n = a_1 \left| F(j\omega_i) \right| \cos(\omega_i t + \phi_i - \phi_d).$$

Here the absolute phase angle has been ignored and only the channel-to-channel difference considered. Under these circumstances the cosine may be used instead of the sine.

The difference between the phase angles ϕ_i and ϕ_d can be determined in terms of the array configuration shown in Fig. B-5.

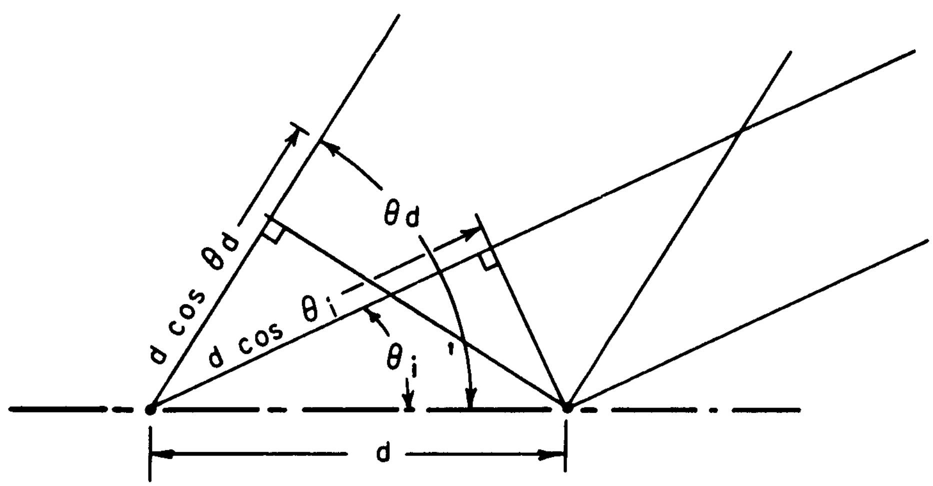


Fig. B-5. Sketch showing geometry for determining the difference in phase delay between adjacent elements for the desired and interfering signals.

It can be seen that this is determined by the difference in path delay from element to element. The difference in phase delay between each successive element is given by

$$\psi_i = \frac{2\pi d}{\lambda_i} \cos \theta_i - \frac{2\pi d}{\lambda_d} \cos \theta_d \tag{B-21}$$

where λ_i and λ_d are the wave lengths of the interfering and desired signals and θ_i and θ_d are the angles to the interfering and desired signals, respectively.

Due to the small percentage difference between interfering and desired signal wavelengths over the phase detector bandwidth (.005% for a 300 cycle phase detector bandwidth at 3 mc) λ_i can be assumed to be approximately equal to λ_d giving

$$\psi_i = \frac{2\pi d}{\lambda_d} (\cos \theta_i - \cos \theta_d) \ . \tag{B-22}$$

The phase difference to the n^{th} element is then $n\psi_i$. The phase ϕ_n is therefore given by

$$\phi_n = a_1 \left| F(j\omega_i) \right| \cos(\omega_i t + n\psi_i). \tag{B-23}$$

The combined array output due to the desired signal is

$$V_d = \sum_{n=0}^{N-1} V_{dn} = \sum_{n=0}^{N-1} e^{ja_1 \left| F(j\omega_i) \right| \cos(\omega_i t + n\psi_i)} \ . \tag{B-24}$$

An expression for the output due to the interfering signal can be obtained from (B-22) simply by including the progressive phase shift from element to element due to the difference in path length. The array output V_i due to the interfering signal is given by

$$\text{(B-25)} \qquad V_i = \sum_{n=0}^{N-1} e^{j[a_1 |F(j\omega_i)| \cos(\omega_i t + n\psi_i) + n\psi_i]} \; .$$

Using the first three terms of the power series expansion for the exponential in Eq. (B-24) gives

$$\text{(B-26)} \qquad V_d \simeq \left(1 - \frac{a_1^2 |F(j\omega_i)|^2}{4}\right) - \frac{a_1^2 \, |F(j\omega_i)|^2}{2} \, \frac{\sin N\psi_i}{\sin \psi_i} \cos[\, 2\omega_i t + (N+1)\psi_i]$$

$$+ \, j a_1 |F(j\omega_i)| \frac{\sin \frac{N\psi_i}{2}}{\sin \frac{\psi_i}{2}} \cos\left[\omega_i t + \frac{N+1}{2} \, \psi_i\right] \; .$$

This result represents amplitude and phase modulation of the desired signal and a reduction in gain. The amplitude modulation is due mainly to the real part and the phase modulation is due to the imaginary part of Eq. (B-26). The gain reduction is associated with the first term of the real part of Eq. (B-26).

A similar result, that is, amplitude and phase modulation and a change in level at the output can be expected for the interfering signal.

List of Symbols for Appendix B

A_d	=	amplitude of the desired signal
A_i	=	amplitude of the interfering signal
A_{in}	=	combined amplitudes of the desired and interfering signals
A_e'	=	combined amplitudes of the desired and interfering signals at the mixer output in the open loop representation
$a_1, a_2, \cdots$	=	Fourier coefficients of phase modulation in the V. C. O. output
b_n	=	binomial coefficients (see (B-12))
d	=	spacing between the elements of the linear array
N	=	number of array elements
$F(j\omega)$	=	open loop transfer function of the phase-locked loop
f_i	=	difference between the desired and interfering signal frequencies $= \frac{\omega_i}{2\pi}$
s	=	$\frac{A_d}{A_i}$ ratio of the desired to interfering signal
s'	=	$\frac{s^2+1}{2s}$
V_d	=	summing network output due to the desired signal
V_{dn}	=	output of the n^{th} channel due to the desired signal
V_i	=	summing network output due to the interfering signal

v_e = mixer output

v_e' = mixer output in the open loop representation

v_{in} = input to the mixer

v_ϕ' = phase detector output in the open loop representation

λ_d = wave-length of the desired signal

λ_i = wave-length of the interfering signal

ϕ_d = phase of the desired signal

ϕ_e' = phase at the output of the mixer due to the desired and interfering signals

ϕ_i = phase of the interfering signal

ϕ_{in} = combined phase of the desired and interfering signals

ϕ_n = phase modulation of the V.C.O. output in the n^{th} channel

ϕ_o = V.C.O. output phase

ω_d = desired signal angular frequency

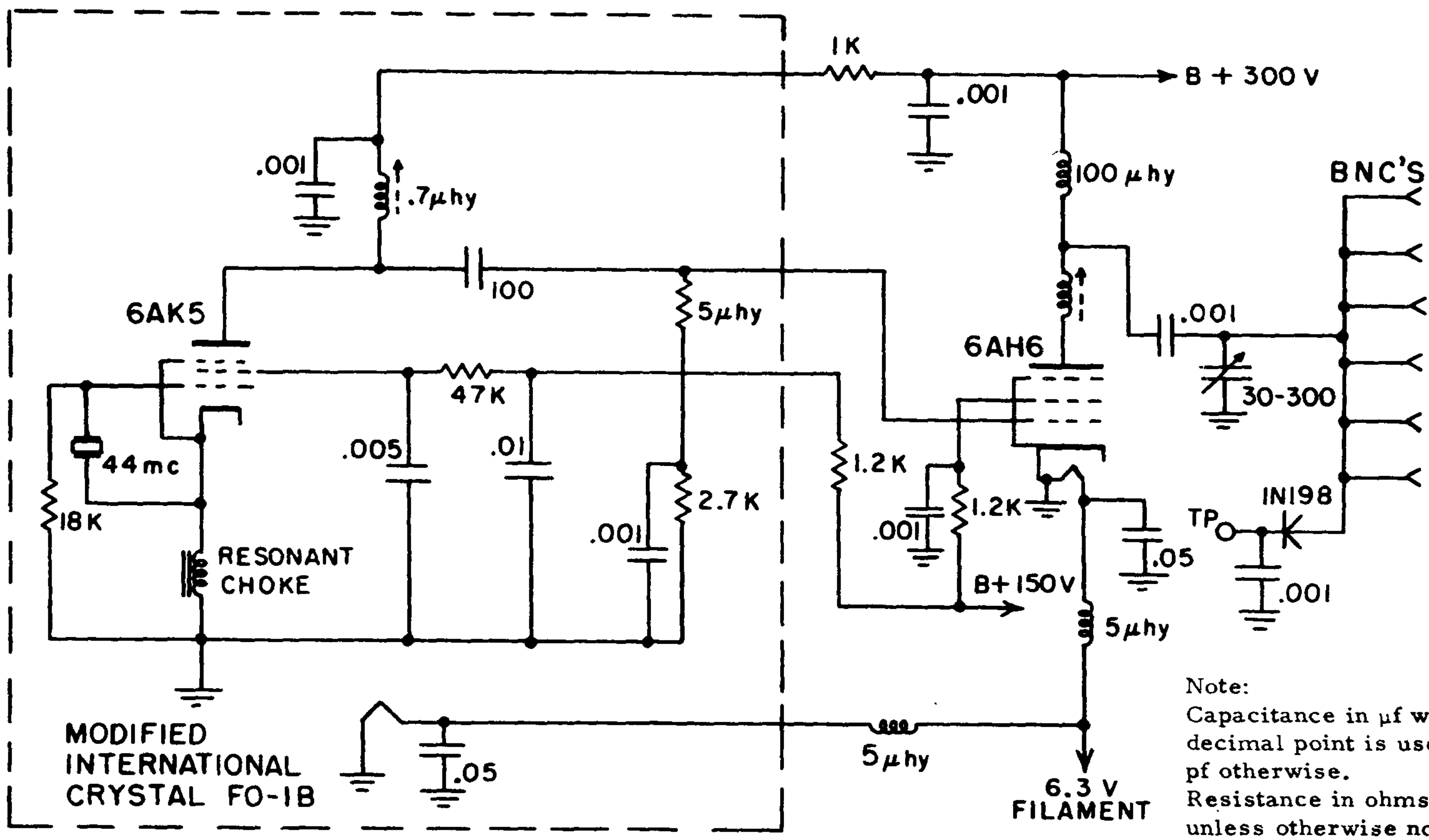
ω_i = difference between the desired and interfering signal angular frequencies

ω_o = V.C.O. output angular frequency

ψ_i = phase difference between the desired and interfering signals at two adjacent elements.

APPENDIX C

CIRCUITS OF THE LOCAL OSCILLATORS AND THE TEST TRANSMITTER



Note:
Capacitance in μf where decimal point is used, pf otherwise.
Resistance in ohms unless otherwise noted.

Fig. C-1. Circuit of the 44 mc local oscillator and amplifier.

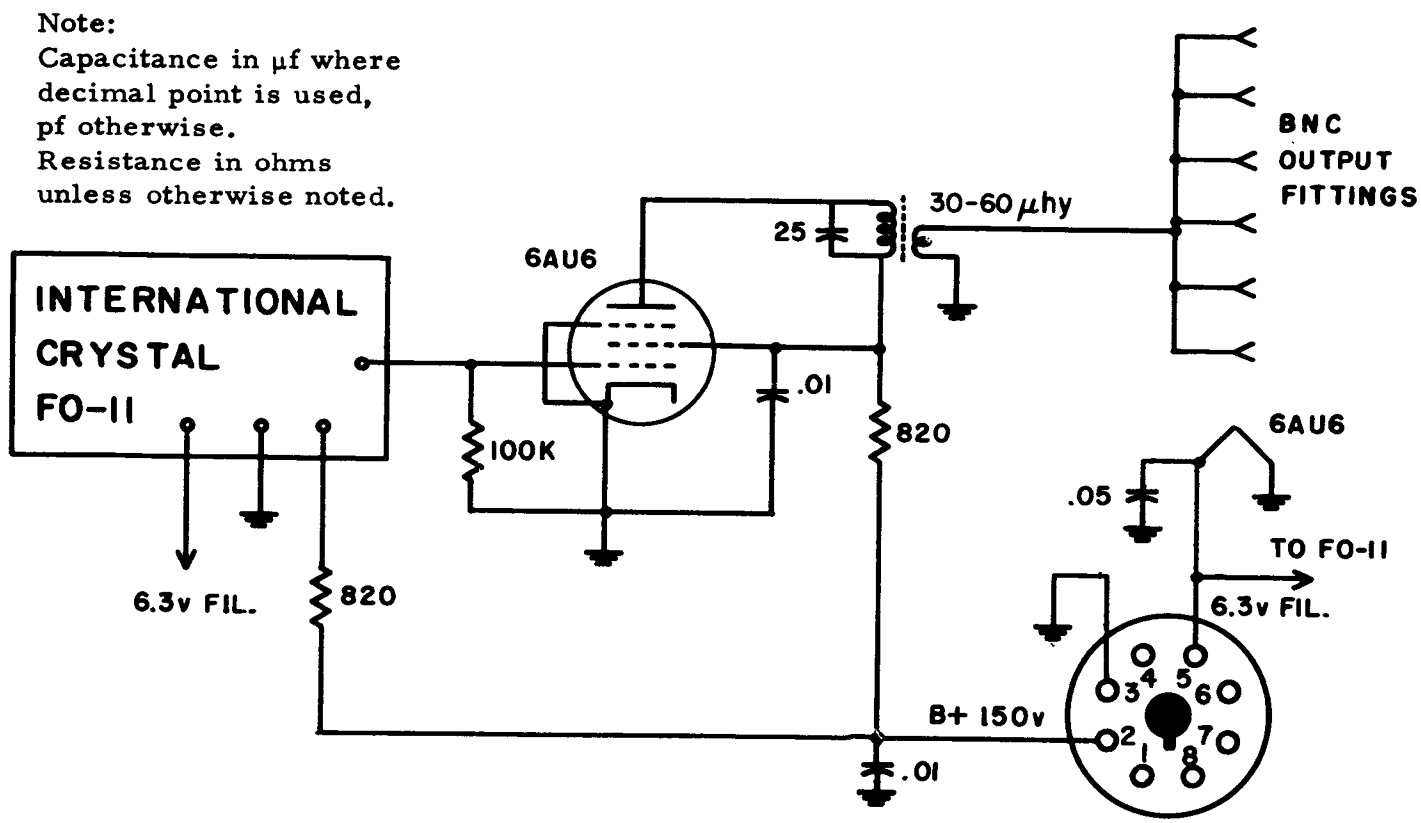
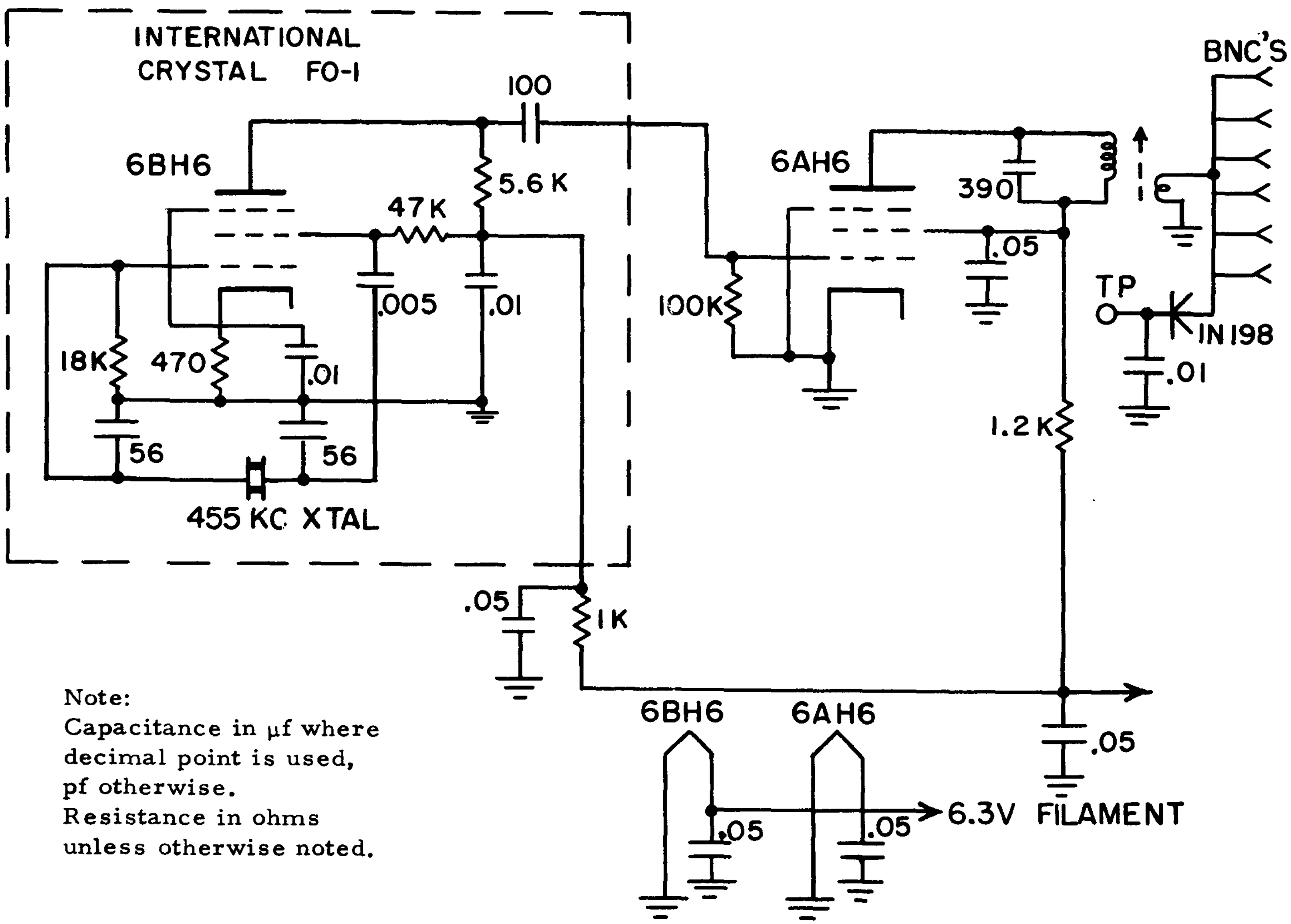
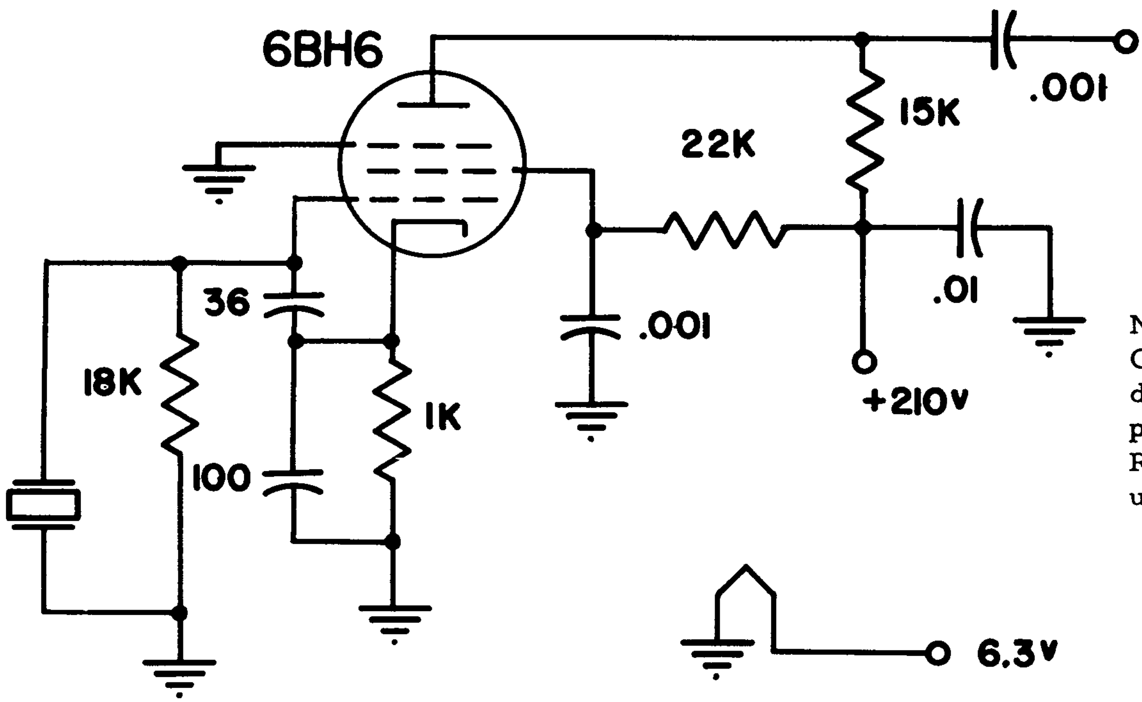


Fig. C-2. Circuit of the 4155 kc local oscillator and amplifier.



Note:
Capacitance in μf where decimal point is used, pf otherwise.
Resistance in ohms unless otherwise noted.

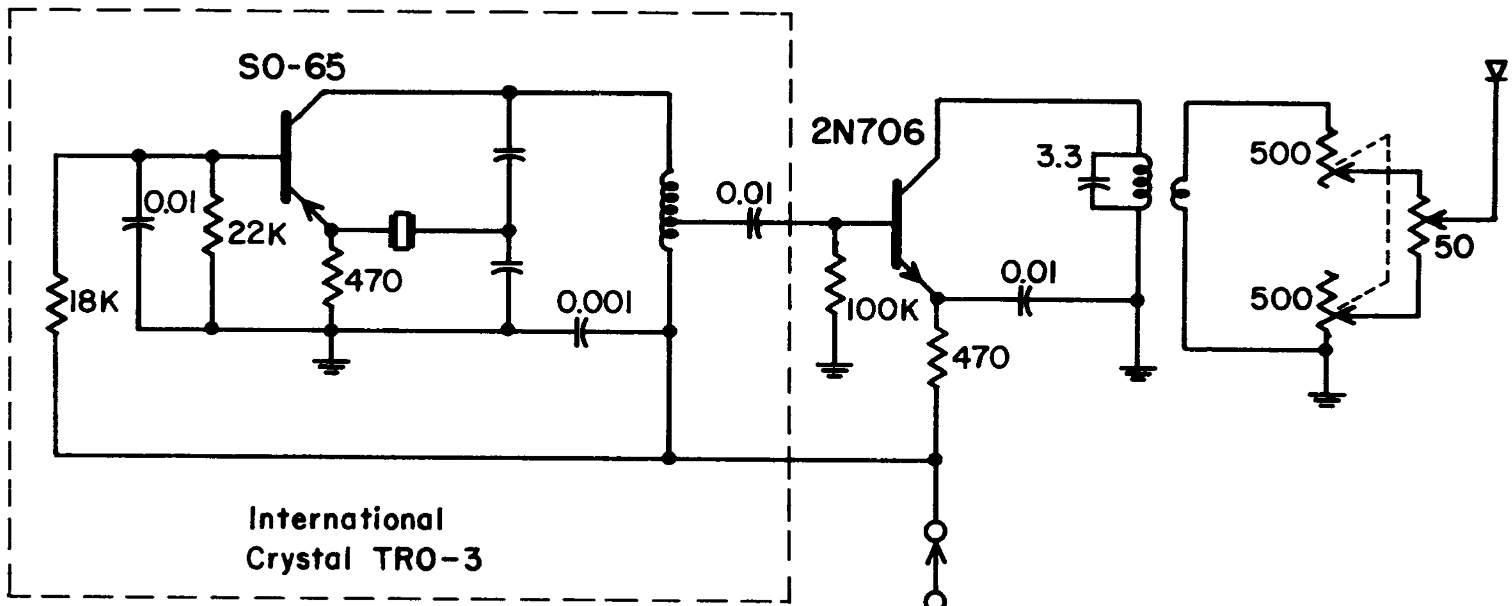
Fig. C-3. Circuit of the 455kc reference oscillator and amplifier.



Note:
Capacitance in µf where
decimal point is used,
pf otherwise.
Resistance in ohms
unless otherwise noted.

Note:
Capacitance in µf where
decimal point is used,
pf otherwise.
Resistance in ohms
unless otherwise noted.

Fig. C-4. Circuit of the International Crystal type FO-11 oscillator used for the V.C.O.'s and for the 4155 kc local oscillator.



Note:
Capacitance in μf where decimal point is used, pf otherwise.
Resistance in ohms unless otherwise noted.

Fig. C-5. Circuit of the desired and undesired signal sources.

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